AMTE

ASSOCIATION OF MATHEMATICS TEACHER EDUCATORS

STANDARDS FOR ELEMENTARY MATHEMATICS SPECIALISTS:

A REFERENCE FOR TEACHER CREDENTIALING AND DEGREE PROGRAMS

DEVELOPED BY THE ASSOCIATION OF MATHEMATICS TEACHER EDUCATORS

The Brookhill FOUNDATION

Association of Mathematics
Teacher Educators

Standards for Elementary Mathematics Specialists:

A Reference for Teacher Credentialing and Degree Programs

Developed by the Association of
Mathematics Teacher Educators

The Brookhill
Foundation

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Preface to the Revised Edition

In 2009, AMTE developed and disseminated the *Standards for Elementary Mathematics Specialists: A Reference for Teacher Credentialing and Degree Programs.* We are now pleased to present the community with an updated version of the document. With this updated version, AMTE continues its commitment to supporting the education and work of elementary mathematics specialists.

This revised version of the *EMS Standards* reflects recent developments in mathematics education, including the Conference Board of Mathematical Sciences' *The Mathematical Education of Teachers II* document as well as *Common Core State Standards – Mathematics* document. The AMTE *EMS Standards* have been updated to align with the recommendations found in *MET II* and describe the content needed to implement *CCSS-M*.

AMTE is grateful for the work of Maggie McGatha of University of Louisville, in leading this revision and to Nicole Rigelman of Portland State University, Terry Goodman of University of Central Missouri, and Joanne Rossi Becker of San José State University for their contributions to the effort.

Fran Arbaugh, AMTE President (2013 - 2015)

Preface to the First Edition

One of the primary goals of the Association of Mathematics Teacher Educators (AMTE) is to advocate or effective policies and practices related to mathematics teacher education. A persistent area of need is the reparation of professionals who are charged with helping young students (particularly in K–6) learn mathematics.

Given the many demands and expertise required to teach all subjects of elementary school (the typical assignment of elementary classroom teachers), AMTE supports the use of elementary mathematics specialists to teach and to support others who teach mathematics at the elementary level. We believe that special expertise is required to do this job well. This expertise includes both a deep and practical knowledge of the content and pedagogy of elementary and middle school mathematics and the ability to work with other professionals to develop their mathematical knowledge for teaching.

It is with pleasure that AMTE offers *Standards for Elementary Mathematics Specialists: A Reference for Teacher Credentialing and Degree Programs*. The *Standards* outline particular knowledge, skills, and dispositions needed by elementary mathematics specialists (EMS). The *Standards* were developed and shared with a broad audience including Conference Board of the Mathematical Sciences (CBMS) member associations and individual AMTE members. Feedback from the public review process was considered and used to produce the final document.

As Chair of the EMS Project Team that produced this document, I am grateful to Jenny Bay-Williams and Francis (Skip) Fennell, past presidents of AMTE, for their encouragement for this work. I also extend thanks to Cathy Kessel who edited the final version of the document and to the Brookhill Foundation for providing support for the effort. In particular, AMTE is grateful to the Foundation's Executive Director, Kathy Stumpf, for sharing her time and insights as the work progressed. Finally, my sincere appreciation goes out to members of the Project Team for their considerable contributions and effort:

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Nicole Rigelman, Portland State University

AMTE invites productive collaborations that are focused on the improvement of mathematics education programs at all levels.

Barbara J. Reys, AMTE President (2009 - 2011)

Standards for Elementary Mathematics Specialists: A Reference for Teacher Credentialing and Degree Programs

Developed by the Association of Mathematics Teacher Educators
Revised October 2013 to align with MET II and to describe content needed to implement CCSS-M.
January 6, 2010

The Association of Mathematics Teacher Educators (AMTE) encourages states to address the urgent need to increase the mathematical knowledge and expertise of elementary school staff by establishing an elementary mathematics specialist (EMS) license, certificate, or endorsement.

Who are elementary mathematics specialists?

Elementary mathematics specialists are teachers, teacher leaders, or coaches who are responsible for supporting effective mathematics instruction and student learning at the classroom, school, district, or state levels. The specific roles and responsibilities of EMS professionals vary according to the needs and plans of each setting. At the classroom level, an EMS professional may teach mathematics to all the elementary students in one or more grade levels or work with a particular group of students to provide remedial or enrichment support services. At the school or district level, EMS professionals may work primarily with teachers as mentors, in a professional development capacity or target school-wide improvement in mathematics. In this role, an EMS professional focuses on strengthening other teachers' understanding of mathematics content and helping them develop more effective instruction and assessment. At the state level, EMS professionals may serve on committees to develop curriculum, assessments and/or policies concerned with mathematics education. Whatever the setting or responsibilities, EMS professionals need a deep and broad knowledge of mathematics content, expertise in using and helping others use effective practices, and the ability to support efforts that help all students learn important mathematics.

Why are EMS professionals needed?

Many have made the case that practicing elementary school teachers are not adequately prepared to meet the demands for increasing student achievement in mathematics (National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008; National Research Council, 1989). In particular, most elementary teachers are generalists—that is, they study and teach all core subjects, rarely developing in-depth knowledge and expertise with regard to teaching elementary mathematics. Wu (2009) describes the situation in this way:

> The fact that many elementary teachers lack the knowledge to teach mathematics with coherence, precision, and reasoning is a systemic problem with grave consequences. Let us note that this is not the fault of our elementary teachers. Indeed, it is altogether unrealistic to expect our generalist elementary teachers to possess this kind of mathematical knowledge. (p. 14)

Further, Wu notes a problem of scale in addressing the situation and suggests utilizing a smaller cadre of well-prepared teachers to focus on mathematics at the elementary grades:

> Given that there are over 2 million elementary teachers, the problem of raising the mathematical proficiency of all elementary teachers is so enormous as to be beyond

> comprehension. A viable alternative is to produce a much smaller corps of mathematics teachers with strong content knowledge who would be solely in charge of teaching mathematics at least beginning with grade 4. (p. 14)

This echoes a statement made 20 years earlier in the National Research Council's *Everybody Counts*:

> The United States … continues to pretend—despite substantial evidence to the contrary—that elementary school teachers are able to teach all subjects equally well. It is time that we identify a cadre of teachers with special interests in mathematics and science who would be well prepared to teach young children both mathematics and science in an integrated, discovery-based environment. (p. 64)

Over the past two decades, others have made similar recommendations (Battista, 1999; Conference Board of the Mathematical Sciences, 2001, p. 11; Learning First Alliance, 1998; National Council of Teachers of Mathematics, 2000, pp. 375–376; Reys & Fennell, 2003). Recently, the National Mathematics Advisory Panel (2008) noted that "the use of teachers who have specialized in elementary mathematics teaching could be a practical alternative to increasing all elementary teachers' content knowledge (a problem of huge scale) by focusing the need for expertise on fewer teachers" (p. 44).

Evidence of impact of EMS professionals

In summarizing their study of reform in schools and districts, Ferrini-Mundy and Johnson (1997) report that the school-based leadership provided by mathematics specialists appeared to be critical to reform. Mathematics specialists "helped spread ideas, facilitate communications among teachers, plan and initiate staff development, and address political problems with administrators and community members" (p. 119).

Recent studies of states with a corps of EMS professionals show evidence of a positive impact on student learning. For example, the Vermont Mathematics Initiative (VMI) has built a corps of K–8 mathematics teacher leaders across the state who can support other teachers in their schools and districts (Kessel, 2009, pp. 36–38). Evaluation studies show evidence that VMI has had a major impact on the teachers themselves, their classroom practice, and students in schools with VMI teachers. Students in VMI schools outperformed those in control schools, and the achievement gap has narrowed between free- or reduced-lunch eligible students in VMI schools and their non-eligible peers in matched schools (Meyers & Harris, 2008). Similar results are emerging from studies in Ohio and Virginia (Brosnan & Erchick, 2009; Campbell & Malkus, 2009).

Examples of state-level EMS certification

Currently 18 states (Arizona, California, Georgia, Idaho, Kentucky, Louisiana, Maryland, Michigan, Missouri, North Carolina, Ohio, Oklahoma, Oregon, Rhode Island, South Dakota, Texas, Utah and Virginia) offer professional designations for elementary mathematics specialists. These programs differ in substantive ways (see The Elementary Mathematics Specialists and Teacher Leaders Project web site, http://www.mathspecialists.org). Some are designed to transition elementary-certified teachers to middle school mathematics teaching assignments. Some focus on broadening the mathematical knowledge of all elementary teachers, while others offer the option for elementary teachers to focus on mathematics teaching as a special area of interest or to prepare for leadership or coaching responsibilities in schools and districts.

Although only 18 states currently offer an EMS license, certificate, or endorsement, experienced teachers in virtually every state are called upon to fill specialized roles related to mathematics teaching in elementary grades, often without special training.

The purpose of this document

The standards in this document outline the knowledge, skills, and leadership qualities necessary for the roles and responsibilities an EMS professional may assume. They are informed by prior articulations of knowledge and skills for teaching elementary mathematics (Ball, Thames, & Phelps, 2008; Conference Board of the Mathematical Sciences, 2001, 2012; Ma, 1999; National Council of Teachers of Mathematics, 2003; National Governors Association Center for Best Practice & Council of Chief School Officers, 2010; Shulman, 1986).

State-level EMS certification provides formal recognition, opportunities, and incentives for teachers to increase their knowledge and skill to teach or to lead others in teaching mathematics in elementary classrooms. With a formal certificate program, school and district administrators will be better positioned to create EMS positions and identify qualified personnel—improving support for their teachers and students.

Articulating the knowledge and skills needed by EMS professionals is a necessary step in initiating state-level certification and program development. The proposed standards are intended to provide a starting point for state agencies to establish certification guidelines. They can also guide institutions of higher education in the creation of programs to prepare elementary mathematics specialists.

Overview of Standards

Prerequisites:

Teacher certification and at least three years of successful mathematics teaching experience.

Program Focus:

At least 24 semester-hours or appropriate equivalent in areas I, II, and III (see pp. 5-8 for further detail). The number of credit hours and proportion of attention to areas I, II, and III will depend on the individual's undergraduate and graduate preparation in mathematics and pedagogy as well as the expertise acquired from teaching and leadership experiences.

I. Content knowledge for teaching mathematics:
   a. Deep understanding of mathematics for grades K–8.[1]
   b. Further specialized mathematics knowledge for teaching.

II. Pedagogical knowledge for teaching mathematics[2]:
   a. Learners and learning.
   b. Teaching.
   c. Curriculum and assessment.

III. Leadership knowledge and skills.

The program should also include a supervised mathematics teaching practicum in which a candidate acquires experience working with a range of student and adult learners including elementary students (e.g., primary, intermediate, struggling, gifted, English language learners) and elementary school teachers, both novice and experienced, in a variety of professional development settings.

I. Content Knowledge for Teaching Mathematics

EMS professionals must know and understand deeply the mathematics of elementary school, the progressions of mathematics topics across several grade levels, as well as how mathematics concepts and skills develop through middle school. This knowledge includes specialized knowledge that teachers need in order to understand and support student learning of elementary mathematics.

a. Deep understanding of mathematics in grades K–8.

Elementary mathematics specialists are expected to acquire the habits of mind of a mathematical thinker and use mathematical practices such as precision in language, construction and comparison of mathematical representations, conjecturing, problem solving, reasoning, and proving. They need to be able to use these practices in the following domains:

<u>Number and Operations</u>

- Pre-number concepts: Non-quantified comparisons (less than, more than, the same), containment (e.g., 5 contains 3), 1-to-1 correspondence, cardinality, meaningful counting, and ordinality.
- A comprehensive repertoire of interpretations, representations, and properties of the four operations of arithmetic (whole numbers) and of the common ways they can be applied.
- Place value: The structure of place-value notation in general and base-10 notation in particular; how place-value notations efficiently represent even very large numbers, as well as decimals; use

[1] Based on recommendations in *Mathematics Education of Teachers Report* (2001, 2012) and *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report (2007).*

[2] Because EMS professionals may work with teachers, their pedagogical knowledge for teaching must apply to teachers-as-learners. Thus, in this section "learners" includes elementary students *and* elementary teachers.

of these notations to order numbers, estimate, and represent order of magnitude (e.g., using scientific notation).

- Multi-digit calculations, including standard algorithms, mental math, and non-standard ways commonly created by students; informal reasoning used in calculations.
- Basic number systems: Whole numbers (non-negative integers), integers, non-negative rational numbers, rational numbers, and real numbers. Relationships among them, and locations of numbers in each system on the number line. Use of standard and non-standard algorithms. What is involved in extending operations from each system (e.g., whole numbers) to larger systems (e.g., rational numbers).
- Multiplicative arithmetic: Factors and factorization, multiples, primes, composite numbers, least common multiple, and greatest common factor.
- Quantitative reasoning and relationships that include rate, ratio, proportion, and rescaling.

<u>Algebra and Functions</u>

- Axioms: Recognize commutativity, associativity, and distributivity, and 0 and 1 as identity elements in the basic number systems; understand how these may be used in computations and to deduce the correctness of algorithms. Understand the relationship between addition and subtraction and between multiplication and division. The need for order-of-operations conventions.
- Algebraic notation, equations and inequalities: Literal symbols, as shorthand names for mathematical objects, or, in the case of numerical *variables*, as indicating an unspecified member of some class of numbers (the "range of variation"). The process of *substitution* of particular numbers into variable expressions. The *solution set* of an algebraic equation or inequality. Transformations of equations (or relations) that do not change the solution set. Solution of pairs of simultaneous linear equations.
- Modeling of problems, both mathematical and "real world," using algebraic equations and inequalities.
- The concept of a function as defining one variable uniquely in terms of another. Familiarity with basic types of functions, including constant, linear, exponential, and quadratic. Representations and partial representations of functions: Formula, graph, table, or verbal description; or, when the variable is discrete, by recursion. Familiarity with functional language such as independent and dependent variables.
- Finding functions to model various kinds of growth, both numerical and geometric.

<u>Geometry and Measurement</u>

- Visualization: Geometric objects are pictured on a 2-dimensional page; for 3-dimensional objects this requires perspective or projection renderings. Producing and reading such representations calls for special skills, both mathematical and drawing.
- Composition and decomposition: A geometric figure can be assembled by joining together various component figures. Conversely, a geometric figure may be decomposed into pieces, for example decomposing a polygon into an assemblage of triangles.
- Congruence and similarity: *Congruence* is the basic concept of geometric "sameness." *Similarity* has to do with rescaling: Two figures are similar if one of them is congruent to a rescaling of the other. For example, all circles are similar, as are all squares and all isosceles right triangles.
- Geometric measurement: a way of attaching a numerical quantity to a geometric figure. Doing this involves a choice of some standard figure (the "unit") and then the measurement is a kind of ratio of the given figure to the unit, or, put differently, how many copies of the unit does it take to compose the given figure? It follows that if a geometric figure is decomposed, then its measure is the sum of the measures of its components. Changing the unit has the effect of multiplying all

measurements by a constant (relating the two units). For example, relating feet to inches, or centimeters to meters.

- Common units of geometric measurement:
  Linear: The unit may be the interval [0, 1] on the number line.
  Area: The unit is a unit square.
  Volume: The unit is the unit cube.
  Angle: Draw a unit circle centered at the vertex of the angle, and consider the arc of the circle cut out by the angle. The *radian* measure of the angle is the length α of that arc. The *degree* measure of the angle is $360\alpha/2\pi$, i.e. 360 times the fraction of the circumference of the circle formed by the arc.
- Basic geometric figures and relationships in each dimension:
  Dimension 1: Line segments, rays, arcs of circles; parallel and perpendicular
  Dimension 2: Polygons, circles;
  Dimension 3: Polyhedral solids, cylinders, cones, spheres.
  Elements of these figures, (e.g., vertex, edge, face). Properties of regularity and symmetry; definitions, names, and classification; various kinds of measurement, and some basic formulas, their relationships, and their rationale (i.e. – perimeter, area, volume); invariance under congruence, and behavior under rescaling.
- Plane coordinates: How they are introduced, and how they support algebraic expression of geometric objects and relationships. Reciprocally, how they afford geometric interpretation of algebraic relations.
- Transformations: Reflections, rotations, translations, dilations, glide reflections; composition of transformations; symmetry and its expression in terms of transformation (e.g., reflection through a line of symmetry); development and expression of congruence and similarity in terms of transformations.
- Geometric constructions; Axiomatic reasoning; Proof: Making and proving conjectures about geometric shapes and relations.

<u>Data Analysis and Probability</u>

- The nature and uses of data: What kinds of questions require data for their answers, and what kinds of data are required? How are relevant data sets created and organized? Designing an investigation, including specification of how the data collected support analysis responsive to the question(s) under investigation.
- Categorical (discrete) data (e.g., gender, favorite ice cream flavor) and measurement (continuous) data.
- Appropriate types of representation of data, and what they afford: For categorical data, relative frequencies. For measurement data, displays of shape, center, and spread.
- Basic concepts of probability and ways to represent them; making judgments under conditions of uncertainty; measuring likelihood; becoming familiar with the concept of randomness; and understanding the relationship between experimental and theoretical probability.
- Conclusions: Understand which representations best support communication of inferences from data, use probability models when appropriate, and account for variability. Understand the limits of generalizability due to non-randomness of a sample population.

b. Specialized mathematics knowledge for teaching. EMS professionals must have mathematical knowledge that enables them to:

- Support the development of *mathematical proficiency* as characterized by conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Research Council, 2001).
- Create opportunities for learners to develop the Standards for Mathematical Practice (National

Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and to critically evaluate learners' selection and use of these practices:.
    - Make sense of problems and persevere in solving them;
    - Reason abstractly and quantitatively;
    - Construct viable arguments and critique the reasoning of others;
    - Model with mathematics;
    - Use appropriate tools strategically;
    - Attend to precision;
    - Look for and make use of structure; and,
    - Look for and express regularity in repeated reasoning.
- Diagnose mathematical misconceptions and errors and design appropriate interventions.
- Decide whether, how and how far, to utilize specific oral or written responses from learners.
- Recognize, evaluate, and respond to multiple, often non-standard solutions to problems.
- Choose and/or design tasks to support the learning of new mathematical ideas or methods, or to test learners' understanding of them.

II. Pedagogical Knowledge for Teaching Mathematics

EMS professionals are expected to have a foundation in *pedagogical content knowledge* (PCK) (Ball, Thames, & Phelps, 2008). This section is informed by and draws upon the 2012 *NCTM NCATE Program Standards for Elementary Mathematics Specialists.*

a. Learners and Learning. EMS professionals must know and be able to:
    - Utilize and build upon learners' existing knowledge, skills, understandings, conceptions and misconceptions to advance learning.
    - Understand learning trajectories related to particular topics in mathematics (e.g., Common Core Standards Writing Team, 2013; Maloney & Confrey, 2013; Sarama & Clements, 2009) and use this knowledge to organize and deliver instruction that is developmentally appropriate and responsive to individual learners.
    - Understand cultural differences among learners (e.g., algorithms or learning practices familiar to different groups of learners) and utilize this knowledge to motivate and extend learning opportunities for individuals and groups of learners.
    - Create social learning contexts that engage learners in discussions and mathematical explorations among peers to motivate and extend learning opportunities.

b. Teaching. EMS professionals must know and be able to:
    - Design, select and/or adapt worthwhile mathematics tasks and sequences of examples that support a particular learning goal.
    - Support students' learning of appropriate technical language associated with mathematics, attending to both mathematical integrity and usability by learners.
    - Construct and evaluate multiple representations of mathematical ideas or processes, establish correspondences between representations, and understand the purpose and value of doing so.
    - Use questions to effectively probe mathematical understanding and make productive use of responses.
    - Develop learners' abilities to give clear and coherent public mathematical communications in a classroom setting.
    - Model effective problem solving and mathematical practices—questioning, representing, communicating, conjecturing, making connections, reasoning and proving, self-monitoring and cultivate the development of such practices in learners.
    - Use various instructional applications of technology, judiciously, in ways that are mathematically and pedagogically grounded.

- Analyze and evaluate student ideas and work, and design appropriate responses.
- Develop skillful and flexible use of different instructional formats—whole group, small group, partner, and individual—in support of learning goals.
- Manage diversities of the classroom and school—cultural, disability, linguistic, gender, socio-economic, developmental—and use appropriate strategies to support mathematical learning of all students.

c. Curriculum and Assessment. EMS professions are expected to:

- Know learning trajectories related to mathematical topics and use this knowledge to sequence activities and design instructional tasks.
- Use multiple strategies, including listening to and understanding the ways students think about mathematics, to assess students' mathematical knowledge.
- Understand the importance of careful sequencing and development of mathematical ideas, concepts, and skills in the preK–middle grades curriculum; be able to engage in discussions and decision-making to establish appropriate benchmarks for learning goals from grades K to 8.
- Select, use, adapt, and determine the suitability of mathematics curricula and teaching materials (e.g., textbooks, technology, manipulatives) for particular learning goals.
- Evaluate the alignment of local and state curriculum standards, district textbooks and district and state assessments, and recommend appropriate adjustments to address gaps.
- Know the different formats, purposes, uses, and limitations of various types of assessment of student learning; be able to choose, design, and/or adapt assessment tasks for monitoring student learning.
- Use the formative assessment cycle (administer a formative assessment task, analyze student responses to the task, and design and reteach lessons based on this analysis) and be able to find or create appropriate resources for this purpose.
- Analyze formative and summative assessment results, make appropriate interpretations and communicate results to appropriate and varied audiences.

III. Leadership Knowledge and Skills

EMS professionals need to be prepared to take on collegial, non-evaluative leadership roles within their schools and districts. They must have a broad view of the many aspects and resources needed to support and facilitate effective instruction and professional growth. This section is informed by and draws upon the 2012 *NCTM NCATE Program Standards for Elementary Mathematics Specialists* and the *Teacher Leader Model Standards* (Teacher Leader Exploratory Consortium, 2011).

EMS professionals must be able to:

- Take an active role in their professional growth by participating in professional development experiences that directly relate to the learning and teaching of mathematics and to their development as a mathematics instructional leader; this may include using professional organization networks, journals, and discussion groups to stay informed about critical issues, policy initiatives, and curriculum trends.
- Engage in and facilitate continuous and collaborative learning that draws upon research in mathematics education to inform practice; enhance learning opportunities for all students' and teachers' mathematical knowledge development; involve colleagues and other school professionals, families, and various stakeholders; and advance the development in themselves and others as reflective practitioners as they utilize group processes to collaboratively solve problems, make decisions, manage conflict, and promote meaningful change.
- Plan, develop, implement, and evaluate professional development programs at the school and/or district level; use and assist teachers in using resources from professional mathematics education

organizations such as teacher/leader discussion groups, teacher networks, and print, digital, and virtual resources/collections; and support teachers in systematically reflecting and learning from practice.

- Evaluate educational structures and policies that affect students' equitable access to high quality mathematics instruction, and act professionally to assure that all students have appropriate opportunities to learn important mathematics. e.g., evaluate the alignment of mathematics curriculum standards, textbooks, and required assessments and make recommendations for addressing learning and achievement gaps; collaborate with school-based professionals to develop evidence-based interventions for high and low-achieving students; advocate for the rights and/or needs of all students and to secure additional resources as needed.
- Use mathematics-focused instructional leadership skills to improve mathematics programs at the school and district levels, e.g., serve as coach/mentor/content facilitator – providing feedback to colleagues to strengthen practice and improve student learning; develop appropriate classroom- or school-level learning environments; build relationships with teachers, administrators and the community; collaborate to create a shared vision and develop an action plan for school improvement; establish and maintain learning communities; partner with school-based professionals to improve each student's achievement; mentor new and experienced teachers to better serve students.
- Select from a repertoire of methods to communicate professionally about students, curriculum, instruction, and assessment to educational constituents—parents and other caregivers, school administrators, and school boards.

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