

Review of
Foundations for Success:
Mathematics Expectations for
the Middle Grades

**By the Association of Mathematics
Teacher Educators (AMTE)**

July 2003

The following review of *Foundations for Success* represents the views of the Association of Mathematics Teacher Educators (AMTE). We established a task force, titled The AMTE Achieve Task Force. All members participated in an individual review as well as a review of this report. In addition, other mathematics educators and the AMTE Board of Directors reviewed the *Foundations for Success* document and the AMTE review of the document.

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AMTE Review of *Foundations for Success*

The following document represents the view of the Association of Mathematics Teacher Educators (AMTE). It was carefully reviewed by a twenty-member representative group of AMTE (referred to here as the AMTE Task Force), other AMTE members, graduate students, and the AMTE Board of Directors. Members of the AMTE Task Force reviewed the document individually, the comments were compiled and each task force member reviewed the summary. The feedback from the second review was incorporated and the result is this document. Many aspects of the *Foundations for Success* document were reviewed positively and considered well-aligned with parallel efforts, such as the *Principles and Standards for School Mathematics* (NCTM, 2000) and various state standards. The task force, however, raised a number of questions and concerns about the purpose, contents, and mathematical sample problems included in the Achieve document. Specific comments are shared below, organized by themes that arose in the reviews.

Purpose

The purpose of Mathematics Achievement Partnership (MAP) is explained and is very extensive. The *Foundations for Success* document appears to be a first step in accomplishing the goals of MAP. In particular, it appears that the purpose of the document is to outline the content expectations for students to know by the end

of 8th grade that are aligned with top performing countries. Given this as the purpose, two fundamental questions were raised:

1. What does this document offer that isn't already offered through the many other recent publications about standards and expectations? The expectations are very similar to NCTM content standards and Mathematical Education of Teachers (MET) standards (Conference Board of the Mathematical Sciences (CBMS), 2001). Is another list of topics necessary? This document provides some sample tasks, but *Principles and Standards* and *Navigations* do this and in a more complete fashion.
2. Is the vision of *Foundations for Success* intended to be a comprehensive vision for middle school mathematics? While it is very similar to the middle school section of *Principles and Standards*, it seems to be more traditional and narrow in focus, excluding processes, such as communication, representation, connections, problem solving, and reasoning. In addition, the focus of content seems to be more procedural than conceptual in nature. As one task force member stated, "the rationale for incorporating an expectation...is its inclusion in mathematics programs of high performing countries; a deeper rationale could be set out, one that includes ...curricular integration and student understanding."

In addition, we offer the following recommendations for clarifying the purpose:

1. The purpose or goals could be stated much sooner in the document. In general the introduction is wordy and repetitive, revisiting topics, such as Third International Mathematics and Science Study (TIMSS), but not providing detail.
2. Who is the audience? On p. 13, the document states that the sample problems are to assist curriculum developers and teacher educators. Teachers are mentioned in various places. The tasks seem to be a resource for teachers.
3. These expectations are described as reflecting "**international standards**." It is not quite clear what these "international standards" are, or how they were actually derived. What was the process in identifying the end-of-eighth grade expectations from other countries? Who was included? While TIMSS is mentioned in numerous places, the details of the process are missing. On p. 16, the document states that "students in other countries routinely attain these goals," but do TIMSS and TIMSS-Repeat (TIMSS-R) results support this statement? Do **most** of students in the high-achieving countries "routinely" meet **all** of these expectations? Or is it the case that most students in the high-achieving countries "routinely" meet **some** of these expectations, but not **all**?
4. Related to #3, what is meant by fundamentals and how are they determined? Are they the intersection or the union of curricula from other countries? For

- example, in the Japanese national course of study, the Pythagorean theorem is not introduced until grade 9. The same is true about quadratic equations/functions, square roots, similar figures, etc. Yet, these topics are included in Foundations for Success as something to know before the end of Grade 8. Details on the establishment of the “fundamentals” are necessary.
5. Rather than preparing “highly skilled workers”, shouldn’t we be preparing “highly skilled citizens”?

An Actionable Vision

Foundations for Success describes this initiative as “the most ambitious effort to date to create an integrated system that enables states to compare achievement and to provide schools with access to world-class training and teaching materials.” A major concern of the AMTE task force is how the MAP goals will be accomplished and how *Foundations for Success* is supposed to be used to enable change. Many questions arose in the review process. How will this document help middle school mathematics teaching and learning? Are the expectations to be a curriculum? If so, more than a list of topics is necessary. TIMSS is cited repeatedly as the rationale for the expectations, yet *Foundations for Success* neglects a well-noted criticism from TIMSS that U.S. curriculum is unfocused and not cohesive. Simply providing a list of expectations to be met at the completion of Grade 8 and sample problems do not provide an actionable vision of how to design a curriculum to effectively meet the expectations listed in *Foundations for Success*.

One of the biggest questions is related to the frequent mention of professional development throughout the document. It is stated that MAP will

help states provide professional development. How will this be done? Support for teachers is also mentioned numerous times. Does this refer to print materials such as this document? What other materials and resources will be provided to teachers?

Another concern about creating an actionable vision is the lack of guidance for the tasks that are provided. There is no information on how the tasks might be used in professional development or in the classroom and the solutions are highly abstract in nature. While experienced teachers may be able to use the tasks without guidance, newer teachers may not be able to modify or incorporate them effectively without guidance. Solutions more representative of how students would solve the tasks would be more beneficial to teachers, curriculum developers, or whoever will be using the tasks.

Nature of the *Foundations for Success* Expectations

As mentioned above, the AMTE task force felt that most of the expectations were aligned with other existing lists of expectations and that they were appropriate. Task force members noted that middle school curriculum has historically been largely repetitious of elementary school curriculum and that different expectations are needed. Many task force members commented that the expectations listed here included their state expectations and NCTM expectations though they were more advanced in certain areas. The algebra strand, in particular, included some expectations that were considered more appropriate

for high school. Specifically, the explorations of quadratic equations and other non-linear functions as cited in the document are more appropriate high school topics. Regarding the geometry strand, memorizing one particular proof such as the Pythagorean Theorem does not seem necessary. Rather than memorize a formal proof, it seems more appropriate for middle school students to develop mathematical arguments. Also, the terminology related to cylinders (p. 29) was questioned: it is defined differently than in *Webster's Dictionary* and in mathematics education resources. The number strand seems to focus more on sets of numbers and the real number system, than on developing number sense for rational numbers and seeing relationships among numbers. The list seems quite traditional. Other specific topics noted as inappropriate for middle school include:

- The attention to primes and factorization seems overemphasized, in particular the uniqueness of factorization seems too advanced.
- Work on significant digits and precision
- Triangle inequality
- Formal work with parallel lines, inscribed angles, properties of spheres, cylinders, prisms, conic sections, arc length, and congruent triangles
- Meanings of constant, variable, and parameter and relations among them
- Fluency with relations such as bank deposit, height and volume of containers. (Why were these singled out?)
- Completing the square
- Rational functions

As mentioned earlier, the question was raised as to whether topics such as quadratic equations and a proof of the Pythagorean Theorem are really in all “high performing” countries. Finally, as mentioned earlier, task force members

mentioned that the expectations were heavily procedural in nature and could be worded such that conceptual development was also emphasized. In other words, the document seems to emphasize form over substance, and facts rather than a flexible use of concepts. *Foundations for Success* states that reasoning is an emphasis, but reasoning does not permeate from the expectations or the sample tasks. Another comment related to expectations is that some are very broad (e.g., demonstrate understanding of the procedures used in computation), while others are very specific, (e.g., identify all two-digit prime numbers).

While the content was in some cases more advanced than other published lists of skills and concepts, there were also omissions that were noted. These include:

- Work flexibly with fractions, decimals, and percents
- Relating number to place value (especially as it relates to expanded notation)
- Develop, analyze, and explain methods for solving problems with proportions
- Knowing everyday situations for using rational number operations
- Inverse proportion (simple rational functions are included)
- Reasoning about data is missing, instead small procedures are mentioned
- Formulating questions that can be addressed with data
- Develop and evaluate inferences and predictions that are based on data
- Randomness and samples
- Strategies for systematic counting
- Any geometry related to using visualization, spatial reasoning, and geometric modeling to solve problems
- Transformations to two-dimensional figures
- Developing relationships among formulas for area and volume
- Estimation (in the number strand)
- Relative rate of growth of arithmetic, geometric, and exponential patterns
- Relationships between variables

One of the greatest concerns of the task force was a lack of a cohesive vision for the expectations. Compared to the NCTM Standards, *Foundations for Success* offers no mention of important processes or principles. *Principles and Standards* calls for a focused and integrated mathematical program. *Foundations for Success* does not prescribe against these expectations, but neither does it incorporate them in its proposed expectations. This was mentioned under “purpose,” but warrants re-mentioning because the task force felt strongly that such processes are critical “fundamentals” for middle school students.

The placement of decimal and fraction computation was questioned. In Appendix A, there is an expectation that students should be able to fluently perform the basic arithmetic computations with decimals and fractions by the start of 6th grade. This counters the *Principles and Standards'* expectation that the emphasis on arithmetic computations with fractions should be in middle school. Moreover, if students entering Grade 6 can “fluently perform arithmetic computation with decimals and fractions,” know that they “are two different representations of the same concepts, and be able to convert among equivalent forms of the same number,” and “understand the concept of the number line and the location on it of integers, fractions, mixed numbers, and decimals,” then what do the middle school expectations such as “order rational numbers and place them on the number line,” “perform accurately manual multi-step calculations...,” “demonstrate understanding of the procedures used in computations,” and “represent rational numbers as fractions or decimals and

translate between these representations” really mean? Is this repetition of elementary topics and if not, in what way are these ideas building on what was learned in elementary school?

A major concern was the lack of use of technology. This counters research that technology supports student understanding and the NCTM technology principle. Computers and calculators, though commonly used in daily life, are neglected in the *Foundations for Success* report. Yet, a number of the sample problems selected are often better solved with the use of technology. We recommend that software and explicit mention of appropriate technology be incorporated into the report, including the sample tasks. Some of the expectations may be more accessible if the use of technology is assumed. This is true across the data, algebra, and geometry strands, where the use of technology could enhance student understanding.

The issue of measurement as a separate strand, or integrated into data and geometry, was debated. A number of task force members felt that if it were to be emphasized, as stated in *Foundations*, then it would be better to have it as its own strand, in particular because this strand is a weakness for U.S. students. Being embedded in two strands has the benefit of showing connections among strands, but this could be true across all the other strands as well.

Across the expectations, the word “understand” is used. Clarification is needed on what “understands” means. *Foundations for Success* defines it as “these interrelated aspects of mathematical proficiency” (p.24). However, so

many of the expectations under “students should understand” seem to be more “conventional” or “factual” ideas that do not involve understanding in the way it is defined by the National Research Council (NRC). Here are some examples:

“the number zero is an integer that is neither negative nor positive”

“irrational numbers are those which by definition cannot be expressed as quotients of integers”

“angles around a point add to 360 degrees and angles on one side of a line add to 180 degrees”

“the sum of the interior angles of a triangles is 180 degrees”

“the conventions for writing algebraic expressions”

Sample Problems

The sample problems raised many issues and varied views among task force members. Here we share some commonly mentioned perspectives. In general, there are some questions about the purpose and use of the problems. As for the quality of the tasks, many task force members felt they were high quality. The quality, however, varies greatly across the tasks, with some being very rich, while others are traditional skill problems. Below we describe in more detail some issues to consider in the revision of these problems.

Purpose. There is concern about the purpose of the tasks. *Foundations for Success* states that the purpose is to, “help illustrate the scope, depth, and meaning of the expectations....They demonstrate the depth of mathematical understanding and reasoning skills that students need...they illustrate at the 8th grade level, many

aspects of mathematical proficiency.” *Foundations for Success* states that the problems are not intended for students (they are “too sophisticated”) but are for teachers. This seems to contradict the purpose statement above stating that they are to illustrate understanding and reasoning skills that students need. Even if they are intended for teachers, if the tasks are not like middle school tasks, then the teacher is left to try to figure out how to translate them into middle school appropriate tasks. This could result in more simplistic tasks or tasks that don’t communicate the same mathematics. One task force member shared this section with a group of teachers and found that they didn’t pick up on the fact that it wasn’t intended for students. These problems may get used “as is” with students, whether that is the intent or not, so the task force recommends that the tasks be revised to be middle school student appropriate.

If the purpose of the tasks is to provide examples for teachers, further explanation is needed related to how they are to be used with teachers. Are they to help teachers understand the content? To help teachers develop their reasoning skills? To help teachers recognize different solution strategies? To provide teachers with examples of the expectations? If the answer to this last question is “yes,” then an explicit connection needs to be made about which problems go with which expectations. If the purpose of these problems is to illustrate the expectations, there should be more discussion on how each problem relates to the expectations. It was not always clear what expectations were being

illustrated by some of the problems. Another suggestion is to integrate these tasks into the expectations sections as exemplars.

One question that warrants attention is how the tasks are related to the assessment that will be developed by MAP. These are not intended to be assessment problems, but if they are the exemplars, shouldn't they be like any assessment questions? The document needs to clarify how these problems differ from the assessments that will be developed.

Quality of Tasks. The problems provided in the document have great potential to illustrate the expectations and to be used in professional development with teachers. But, more guidance is needed in both of these areas. One stated purpose (quoted above) was to illustrate student reasoning. A number of task force members felt that many of the tasks did not involve reasoning at all. While there was variation on how many problems the reviewers liked, there was general consensus that there are a mixture of rich problems and trivial, uninteresting problems. As one task force member noted, "some activities border on 'trivial' or solely definitional. Others are commonplace, even 'traditional' in scope and structure and do not serve to take the learner to new perspectives. Still others are creative and require a unique or unconventional view of the concept." Task force members advocate for more of the latter type of problem and fewer of the first two types. For example, problems N4 and N7 were cited numerous times as uninteresting and rote (as were others in the number strand). The other strands have a mix of dry problems with some very interesting ones. G14, for example,

was a rich problem with multiple solutions available and A20 and A21 include contexts that are interesting to middle school students. Some of the problems were considered too advanced for middle school students. This could be because they are intended for teachers, but if the tasks are aligned to expectations, they should be appropriate middle school problems. Examples of these include: G4, G8, A16, and A19. Too many of the problems seemed to be solely procedural and look like traditional textbook problems. Many AMTE Task Force members noted that context needs to be added to many of these tasks to make the problems more meaningful and interesting.

Finally, and importantly, there are numerous errors in this section and a very careful edit is necessary.

General Comments and Recommendations

Many of the questions that arose from reviewing *Foundations for Success* can be summed up in the word “how.” How will teachers be supported? How will states be supported? How will assessments be developed and curriculum developed? How will these expectations be implemented? How will you disseminate and ensure use of the document? How will Achieve efforts address diversity? It is implied by the introduction that answers to the questions may be forthcoming, but this particular document does not address these questions and therefore leaves the reader with (1) a skills/concept list and (2) some sample

problems. Here we summarize important recommendations that have been shared earlier.

- Heighten the use of processes, such as communication and reasoning.
- Include problems that use technology and other learning tools (e.g., manipulatives).
- State target audience at the very start of the document.
- Remove the repetition in the document. The introduction can be stated more succinctly and clearly. The document is repetitive in that it repeats within its pages, and also because it repeats what other reports have already established. Address clearly the answer to, “What does this offer that isn’t already in print through CBMS or NCTM?”
- The document would be much more powerful if it offered guidance on how to move schools and teachers toward the expectations. Listing topics to know by the end of 8th grade is not very useful without this guidance.
- Attention should be given to instructional strategies. Suggestions can be made with the sample problems. Having samples of a variety of student solutions to the problems would also improve their usefulness.
- Attend to how the process will begin in middle school when elementary students do not have the prerequisite knowledge listed in Appendix A. Explicitly address strategies for making the transition. Strategies or steps for “raising the bar” would be helpful.
- Include a rationale for why these expectations help middle school students develop mathematically.
- Provide explanations with each sample problem that include the targeted expectations and instructional strategies that might be used. Including a grade level recommendation and/or prerequisite skills would be helpful as well.
- The conclusion would be an appropriate place to begin to outline a professional development plan that operationalizes the vision presented within.
- The sample problems that are selected must all be high quality.

The AMTE Task Force hopes this review is useful and will impact the revisions of *Foundations for Success*. AMTE is committed to having middle school curriculum and instruction that is “world class” and welcome future opportunities to collaborate with the Mathematics Achievement Partnership.