

## STEM Integration via Matrix Multiplication and Transformations

*David Glassmeyer, Kennesaw State University*

*Binita Patel, Fulton County STEM Magnet High School*

Both the Common Core State Standards for Mathematics (CCSSM) and the Next Generation Science Standards (NGSS) challenge teachers to connect STEM concepts in deeper and more meaningful ways. While many teachers are interested in using integrated STEM approaches, teacher education efforts need to support teachers to understand and implement STEM connections (Shernoff et al., 2017). Specifically, “training teachers who are well versed in multiple content fields and are able to engage students in such active learning environments is the task set before teacher education” (Enderson et al. 2020, p. 350). This article shares how secondary teachers explored mathematics and science concepts (specifically matrices and computer animation) and how one teacher then integrated a related STEM lesson with her high school students. We overview the lesson implemented in the graduate program, explain the related K-12 lesson implemented, and provide insight into how these computer science and mathematics concepts create integrated STEM opportunities in secondary mathematics.

### Graduate Lesson: Using Matrix Multiplication to Transform Points

As a mathematics teacher educator, I (Glassmeyer) work in graduate programs for secondary teachers to develop teachers' content knowledge of mathematics. I strive to integrate STEM concepts so teachers can use these ideas with their K-12 students. One content course includes the objectives of deepening teachers' understanding of matrix operations and illustrating linear algebra concepts using matrices and transformations. I created the following lesson to develop these understandings by integrating computer science concepts of animation and the transformation of points. While there are whole courses and textbooks integrating these topics, I make use of limited time by having teachers explore the basis of animation and computer-generated images: transforming points with matrices.

First, teachers use [this Geogebra sketch](#) to explore the five types of transformations on the letter F, showing how the shape's 10 points transform about the coordinate plane. Second, teachers answer the questions in Figure 1, which includes finding the operation matrix that, when applied to the input value matrix, produces the output value matrix resulting from a vertical reflection across the y-axis.

1. Use the GeoGebra file to determine the following transformation of the points after Reflection #1 (<https://ggbm.at/adTxUmXt>)

operation matrix

	A	B	C	D	E	F
x-coordinate	2	2	5	5	3	3
y-coordinate	1	5	5	4	4	3

input values

2. For a generic point P on the figure, create a function  $f(x)$  that inputs the original  $x$ -coordinate and outputs the  $x$ -coordinate after a reflection over the  $y$ -axis.

3. For a generic point P on the figure, create a function  $f(y)$  that inputs the original  $y$ -coordinate and outputs the  $y$ -coordinate after a reflection over the  $y$ -axis.

	A'	B'	C'	D'	E'	F'
x-coordinate						
y-coordinate						

output values

- 4. Determine the operation matrix that will transform our points as given in Transformation #1.

Figure 1: Teachers created functions and an operation matrix to vertically reflect points across the  $y$ -axis.

Small groups of teachers present their solutions, which include:

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 2 & 2 & 5 & 5 & 3 & 3 \\ 1 & 5 & 5 & 4 & 4 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -5 & -5 & -3 & -3 \\ 1 & 5 & 5 & 4 & 4 & 3 \end{pmatrix}$$

operation matrix \* input matrix = output matrix

After discussing the operation matrix for a vertical reflection across the  $y$ -axis, teachers create operation matrices for a horizontal reflection across the  $x$ -axis, a dilation by a factor of their choice, a rotation by 90 degrees counterclockwise, and a rotation by a degree measurement of their choice. A similar questioning structure to Figure 1 scaffolds teachers' construction of the operation matrix from the functions ( $f(x)$ ,  $f(y)$ ) for each operation. Additionally, we discuss how the rotation of 90 degrees can help construct an operation matrix for any degree rotation. To conclude the lesson, we discuss how movies, GIFs, and cartoons are sequences of images that change from frame to frame, and how the images are constructed of pixels (or points of color) that change color values or position. Creating efficient ways to transform those points is of

relevance to computer animation and provides motivation for why operation matrices should be constructed to transform as many points as needed to create the kind of special effects and shows we regularly enjoy.

After implementing the lesson, one of the teachers in the class (Patel) informed me she had used this information in her high school classroom and got me thinking of how learners can more deeply explore the intersection of matrix multiplication and transformation of images.

### Transforming Images with Matrix Multiplication

While matrices can organize information about the coordinates of a point, they can also hold details about that specific point on an image. Images are comprised of pixels, which directly store information about their color and indirectly store information about where they are located on an image. Image transformations can more easily be explored by editing an image of black and white pixels in some shape. Using the number zero to denote a white pixel and the number one to denote a black pixel, the color values for each consecutive pixel can be organized into a matrix. By focusing on a specific pixel, as illustrated in Figure 2, learners can determine a new gray-scaled color value by averaging the color values of the given pixel and those surrounding it.

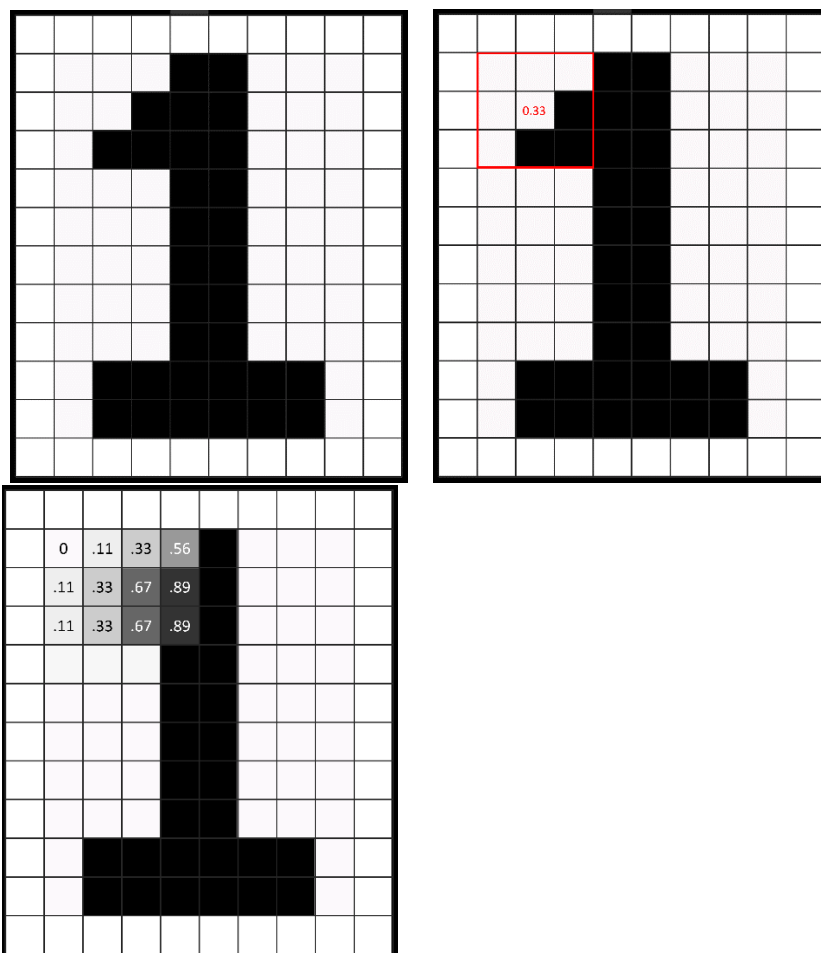


Figure 2: Matrices can be used to transform a black and white image into gray-scaled values for each pixel by averaging the nine surrounding color values.

Having teachers use matrices to blur an image illustrates the connections between mathematics and computer science. This illustrative format invited Patel to consider how computers calculate and process image transformations that we recognize visually on the screen.

### K12 Student Lesson: Transforming Images

At a new STEM high school, I (Patel) teach in both the mathematics department and the career technology department, focusing on Information Technology. The school focuses on cross-curricular opportunities with an emphasis on Project-Based Learning. During the exploration unit of the AP Computer Science Principles course, students were challenged to apply the coding skills they learned throughout the first semester to develop an interactive program in which they structured code segments into a series of functions. Several students based their program on content from different subjects; for example, students in Chemistry created interactive review quizzes and study guides to help their peers prepare for assessments. Other students created an integrated project

experience, developing a computer science component to supplement their current science project that utilized the scientific method.

I helped one student whose project focused on using a Python program to edit an image by transforming its pixels. He wanted to explore how a computer processes an image to appear blurred to the human eye. My background in computer science and exposure to integrating matrix multiplication with image transformation in my graduate coursework helped me form cross-disciplinary questions to guide the student's curiosity. We began by drawing connections between the mathematical concept of matrix multiplication and the programmatical concept of arrays in order to view geometric transformations from two different lenses. Though the student easily accepted the synonymy between matrices and arrays, I noticed the conceptual understanding that was necessary for him to couple the terms.

His program's blur function stored the color values for each pixel in the pre-image based on the amount of red, green, and blue (RGB). The color values and pixel locations served as input values for his blur function, which organizes information into arrays. Analogous to the gray-scaled blur in the graduate course activity, each pixel in a photo can serve as a center point for a square-shaped section referred to as a kernel. In image processing, the student determined that the RGB values of all pixels encompassed in the kernel can contribute to the pixel's new RGB value in the edited image. Figure 3 shows an example kernel in which the color of the sky in the image originally appeared clearer but adopts a blur when the color values are averaged together.

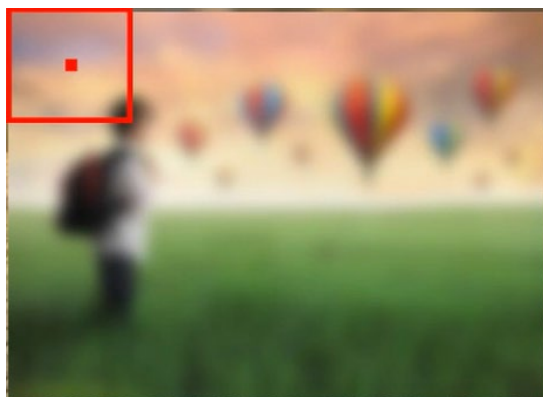


Figure 3: The kernel is defined by a center pixel and the pixels included in the surrounding square with amplitude,  $A$ .

The student's program managed three user-input arguments for a function: the input file of the image, the name for the output file of the image being processed, and the kernel size which defines the degree of the blur. Similar to how the mathematical format of organizing values can employ matrices, the computer science setting requires a two-dimensional array with rows and columns in order for the values to maintain their relationships. He structured the code using an array in which each index position

specifies the numerical position of a value or set of values. In his case, the set of RGB values for each pixel is stored in the index position that represents the x- and y-coordinates of the pixel, collocating the color values and position.

What is noteworthy about the student completing this project is that even though he is interested in learning more advanced mathematics, he was in an accelerated Geometry and Algebra 2 course. As his IT teacher, I admired his inquiry of Calculus of Statistics as he worked to transform the pixels of an image mathematically. When providing him support, I would begin by having him work with a more rudimentary function; a more direct mean blur like the gray-scaled number 1 would allow a greater number of students to apply their prior knowledge of mathematics and achieve a similar blur effect on an image. Investigation of a subtle mean blur and a more intensive perspective blur led the student to consider a different, more average blur based on the Gaussian function. The bell-shaped curve of the Gaussian function considers the weight of each pixel in the kernel such that the pixels closer to the center are more heavily weighted than the pixels towards the perimeter of the square. By the end of the project, the student had mapped each variable from the Gaussian function as an input, or separate parameter, for the function. By altering the amplitude,  $A$ , of the kernel, the student realized he could easily adjust the strength of the blur. You can see the student share what he learned and more about this process in [this video](#) he created for a separate course on Audio-Video Technology Film.

My student's greatest obstacle was that the advanced mathematics added a level of complexity to his programming-focused exploration. The interrelatedness between matrices and arrays might lend itself to extensions students could explore, such as the rotation of images to any degree. While the student had not priorly seen matrices nor trigonometric functions in his mathematics courses, he became more inherently interested in learning their application to operation matrices for transformations. This student's goal is to develop a fully functional image-editing application from his current blur function, the code for which he uploaded online [here](#).

## Reflective Conclusion

I (Glassmeyer) was impressed with the extent to which this high school student's project integrated matrices and image transformations. The depth of this high school student's exploration exemplifies how much potential still exists for me to integrate STEM concepts of matrices, image editing, and computer animation. I plan on advancing my content knowledge of this topic, then extending the matrix multiplication lesson to include blurring examples through a related homework assignment for my teachers that has them explore these ideas and see examples like the one presented in this article about how high school students can and do use integrated STEM concepts. I will also share this student's video to show other teachers what they may encounter in the classroom.

Engaging in activities from the student perspective as a mathematics educator inspired me (Patel) to create various supplemental materials – mostly digital – in my own classroom setting. Though I began by simply drawing connections that I could share during instruction, I learned that students became self-motivated to extend their mathematics education through applications in new or existing projects in which they became invested. In the growing digital age, mathematics provides the basis for the computation behind computer science programs. Teaching courses in both the mathematics and computer science departments, while challenging for creating depth in lesson content, has been a rewarding experience for building a breadth of instructional materials across disciplines. With the development of meaningful and accessible learning targets, I found that a broad yet relevant project idea can inspire students to strive for higher levels of learning.

## References

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