

Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education

A Task Force Report for the
Association of Mathematics Teacher Educators

Forward

This report, in some ways, has been several years in the making. As you will see, the thinking behind *Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education* began with a national conference on doctoral programs in mathematics education in 1999. I know all about this conference, which was organized by Robert Reys and Jeremy Kilpatrick. You see, the thinking behind this work actually began before 1998 with a conference proposal submitted to this then National Science Foundation Program Officer. The conference was important, the work that followed by Reys and Kilpatrick in the editing and publication of *One Field, Many Paths: U.S. Doctoral Programs in Mathematics Education* (CBMS, 2001) may be more important. This manuscript, completed by a task force representing some of the finest and most well-respected mathematics educators in the country, provides the Association of Mathematics Teacher Educators (AMTE) with the concluding effort of this now four-year project.

Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education is not a mandate. This document should serve to drive discussion and action as institutions of higher education consider reviewing, revising, or creating doctoral programs in mathematics education. The task force and I believe that this initial AMTE contribution is best used to help define and discuss the core elements of doctoral study in mathematics education.

We hope the dialogue that this publication generates is helpful to institutions and individuals as we all consider the elements of doctoral study in mathematics education.

In closing, allow me sincerely thank the Task Force for this important and initial AMTE publication.

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Part 1: Background

The 1999 National Conference on Doctoral Programs in Mathematics Education funded by the National Science Foundation revealed great diversity in goals, components, and expectations of mathematics education doctoral programs across the United States (see Reys & Kilpatrick, 2001). The number of and variation in programs was further documented with data from the National Research Council's *Annual Survey of Doctoral Recipients in the United States* (NRC, 2000), which reported that from 1980 to 2000, over 120 different institutions awarded doctoral degrees to people identifying their major discipline as mathematics education. Two related and nontrivial questions surfaced during the conference: Is there a "core or canon of knowledge" needed by those earning doctorates in mathematics education? And what are the essential elements of a doctoral program in mathematics education?

Identifying a common core of knowledge for recipients of doctorates in mathematics education is confounded by several factors, including the following:

1. Mathematics education is a young field, with most programs having evolved during the last 50 years.
2. In some universities, doctoral programs are located in departments of mathematics; in others, they are in schools or colleges of education.
3. Careers awaiting people with doctoral degrees in mathematics education vary greatly. Jobs include positions as classroom teachers and district or state mathematics supervisors, as well as employment by textbook publishers, test development enterprises, research centers, and institutions of higher education.
4. Positions in higher education vary greatly in focus and scope.

Nationwide, there is an acute shortage of people with doctorates in mathematics education to fill positions in universities granting advanced degrees in the field (Reys, 2000, 2002). That shortage is exacerbated when, as happens increasingly, new degree recipients take other types of jobs. Over half take faculty positions in higher education, but these positions are often in junior colleges and four-year institutions and not just in institutions with doctoral programs (Glasgow, 2000). Others take jobs outside higher education.

Those taking faculty positions in higher education may be called upon to teach undergraduate and graduate level mathematics courses in departments of mathematics. Those in schools or colleges of education typically undertake some combination of teaching undergraduate methods courses in mathematics education, supervising student teachers, teaching graduate courses in mathematics education, and providing professional development activities for in-service teachers of grades prekindergarten to 14. Although teaching expectations vary, positions in higher education typically require the establishment of an active research agenda.

Regardless of the job, a knowledge base is central to the work of mathematics educators. If mathematics education is to advance its status as an emerging discipline and effectively prepare graduates for a wide range of positions, it is important to deliberate about "essential

elements of a doctoral program in mathematics education.” A rationale for the mathematics education community to address this issue was provided in the closing chapter of *One Field, Many Paths: U. S. Doctoral Programs in Mathematics Education*:

The absence of system-wide standards for doctoral programs is perhaps, the most serious challenge facing systematic improvement efforts. Shared standards have never existed for U.S. programs in mathematics education. . . Indeed, participants in the system have grown accustomed to creating their own standards at each local site. Developing a consensus on goals or standards is a significant step because it will require a change of practice. It will remove some of the isolation and autonomy of individual programs in favor of a shared commitment to improving the system of doctoral education. Changing practice in this way involves changing culture, and cultural changes are neither quick nor easy. (Hiebert, Kilpatrick, & Lindquist, 2001, p. 155)

Consensus on the core elements of doctoral programs in mathematics education will be difficult (some might say impossible), but progress can be made if educators engage in an iterative process involving thoughtful discussion and reflection. The process itself has the potential to improve the quality of doctoral programs in mathematics education, the preparation of graduates of these programs, and ultimately the profession of mathematics education.

To move this process forward, the Association of Mathematics Teacher Educators (AMTE) appointed a Task Force charged with identifying “core elements of quality doctoral programs in mathematics education.” AMTE recognizes that the challenge is formidable, but its action was based on the belief that articulation of core elements would inform institutions initiating new doctoral programs in mathematics education as well as institutions with established programs.

This report is organized into three parts:

Part 1 - Background

Part 2 - Core Knowledge Expectations for Doctorates in Mathematics Education. Each element is identified, together with a brief rationale for its inclusion, and some means of addressing the elements is briefly described.

Part 3 - Institutional Capacity Needed to Support Quality Doctoral Programs. A list of institutional components necessary to support high quality doctoral programs is proposed.

The Task Force believes that the quality of doctoral programs in mathematics education is influenced by the availability of opportunities to acquire an essential knowledge base. By detailing these components, we hope to serve at least two audiences: faculty and future doctoral students. Consequently, this report might be used by faculty to review existing programs in light of these core elements and to make changes as deemed appropriate. Students might use the information to make informed choices about which institution to enter to pursue a doctorate in mathematics education.

This report builds on discussions of key components of doctoral programs in mathematics education found in *One Field, Many Paths: U. S. Doctoral Programs in Mathematics Education* (Reys & Kilpatrick, 2001). Because of its brevity and focus, the report can only point toward a direction we believe will lead to an overall improvement of doctoral programs and their graduates in mathematics education. The report should encourage faculty involved with the education of doctoral students to reflect on their program and use this reflection to improve its quality. We also recognize that a number of institutions are contemplating the establishment of doctoral programs in mathematics education. The report should be useful to that group as they set about developing high quality doctoral programs.

Part 2: Core Knowledge Expectations for Doctorates in Mathematics Education

A goal of doctoral programs is to develop leaders capable of contributing to the profession of mathematics education and communicating knowledgeably about many topics and issues in mathematics and mathematics education. The following section outlines “core knowledge” that the Task Force believes is essential for the work undertaken by most graduates of doctoral programs in mathematics education. It is organized around eight areas of inquiry. These areas should be viewed not as equivalent to courses but rather as the knowledge base doctoral students should acquire through a range of experiences. Such experiences should include courses as well as seminars, clinical experiences, internships, assistantships, and independent study.

Mathematics Content

Mathematics educators need broad and deep mathematical knowledge both to identify the big ideas in the pre-K–14 mathematics curriculum and to examine how those ideas develop throughout the curriculum. Regardless of the entering level of mathematical knowledge they bring to a doctoral program, students should continue to study mathematics while in the program. Although each student may follow a different program of study, all should exit the program with some graduate study of mathematics and a deep and broad understanding of pre-K–14 mathematics. Standard courses in advanced mathematics are appropriate for students pursuing some goals, but such courses are seldom consciously designed or delivered in ways that enhance the knowledge or understanding of pre-K–14 mathematics. Avenues to accomplish broad understanding could include combinations of the following: formal mathematics course work, special courses or seminars examining specialized (pre-K–14) mathematics from advanced points of view, and clinical experiences in curricular development with intense scrutiny of the interconnectedness of different mathematical strands.

For mathematics educators, how one knows mathematics is vital. And how one knows mathematics is a function of how one comes to know mathematics. Knowing mathematics as a teacher needs to know it must be attended to within doctoral programs in mathematics education.

Research

Research as the hallmark of a doctoral program demands that mathematics educators critique research reports; synthesize results; interpret research findings for practitioners; and design, carry out, report, and direct research studies. Doctoral programs must prepare graduates to conceptualize and conduct research that advances the field's understanding of mathematics learning and teaching and to communicate the results clearly to a variety of audiences. The complexity of questions related to mathematics education demands multiple methodologies and the ability to choose among methods and to design studies and analysis techniques appropriate to the question under study. Graduates should understand and be able to apply general methods of inquiry and should have acquired expertise in both quantitative and qualitative research methodologies. Use of technology as a research tool is a vital part of preparing researchers. In addition, graduates should be well grounded in current and historical research in the field of mathematics education. Competence as a researcher is gained from early and continuous opportunities to read, interpret, use, and conduct research throughout the doctoral program. Doctoral programs should emphasize apprenticeship training in addition to formal coursework focused on research.

Historical, Social, Political, and Economic Context of Education

Schools operate in a social context that is influenced by a variety of historical, social, political, and economic factors. Mathematics education is not immune to the forces shaping education in general, so the mathematics educator needs to understand those forces and how they work. For example, knowing the historical evolution of the equity movement equips the mathematics educator better to understand and respond to current efforts to address inequities.

Teaching, learning, assessment, technology, research, mathematics, and curriculum all have a history. Knowledge of these histories and their interrelationships provides a valuable lens to interpret, understand, and act upon these areas and to participate in the process of improvement. Studying the history of mathematics education is an integral part of a doctoral program. This knowledge includes, but is not limited to, policies that have influenced and shaped the evolution of mathematics education. Familiarity with reports from major commissions, committees, and professional organizations is a prerequisite to understanding and responding to contemporary debates about the status and progress of mathematics education.

Learning

Fundamental theories of learning mathematics provide the foundation for thinking about issues in mathematics education. Mathematics educators need to understand these theories and the distinctions among them in terms of both the kind of learning they are trying to explain and the theoretical constructs that have proven useful over time. A treatment of both historic and contemporary theories of learning should be a part of all doctoral programs in mathematics education.

Drawing on current theories and research, doctoral students should understand how people of different ages, mathematical backgrounds, and aptitudes learn mathematics. This

understanding may be accomplished by various means including courses, seminars, or special readings focusing on theories of learning and the accompanying research evidence. In addition, a doctoral program should provide opportunities for candidates to link their knowledge to practice in designing or evaluating curricula, setting learning goals, and creating cognitively appropriate patterns of instruction.

Teaching and Teacher Education

For those students who are preparing to become teacher educators, mathematics education doctoral programs should provide mentored clinical experiences that develop expertise in designing and teaching mathematics content and methods courses for teachers, supervising field experiences of prospective teachers, and organizing professional development experiences for practicing teachers. Given the centrality of teaching to the work of mathematics educators, it is important that recipients of doctorates be knowledgeable about research on teaching and teacher education, and capable of contributing to that body of research. They should also be able to help teachers acquire knowledge of research on teaching and translate it to their own practice.

For those involved in teacher education, substantial teaching experience is recommended for entry into the doctoral program. Candidates without substantial teaching experience should acquire such experiences part of the doctoral program. To provide guidance to prospective and in-service teachers, mathematics teacher educators must themselves be critically reflective about their own teaching. They should be confident and competent in choosing and using effective instructional strategies consistent with mathematics learning goals as well as adapting and, in some cases, developing curriculum and assessment materials that facilitate student learning. Developing increased teaching competence can be accomplished by experiences such as supervised teaching, team teaching, and mentoring by highly skilled teachers.

Technology

Technological tools are vital to the development of mathematical concepts and processes, and their availability is changing mathematics at all levels. Consequently, mathematics educators need both knowledge of and an ability to use such tools effectively. Graduates of doctoral programs in mathematics education should understand and be able to utilize technology as a tool of inquiry that has implications for teaching and learning mathematics. Although technology offers opportunities to present and explore mathematics in new ways, it is critical that doctoral students understand the potential and limitations of technology. They should be able to design learning experiences for students and teachers at various levels that utilize technology to enable and support mathematics exploration and learning. Fluency is expected with technology tools that support teaching, learning, and research. Knowledge of research related to the interaction of technology and mathematics teaching and learning should be a specific focus of study within a doctoral program.

Curriculum

The work of mathematics educators involves designing effective curricula and learning environments to facilitate the development of deep and connected mathematical

understanding. To do such work, doctoral students need experiences in curriculum analysis, design, and evaluation. For example, they need to understand the role and influence of local, state, and national curriculum frameworks and standards on the design and implementation of school programs. Curriculum development is informed by knowledge of current theories and research about human learning, how to connect different areas of mathematics, and how students come to appreciate mathematics as a discipline. It is also informed by knowledge of how curricula, technology, and instructional strategies work together to support mathematics learning. Avenues to develop a deeper knowledge of curriculum and curricular issues include studies of different strands of curricula, comparisons of international curricula, and studies of mathematical concepts across grade levels. Evaluation of curricula should include experiences in examining topics and making judgments about their relative importance in the curriculum and their utility in developing other mathematical ideas.

Assessment

Mathematics educators must have knowledge of assessment. Doctoral graduates should know the literature on assessment, including the major influences assessment practices have on the intended, implemented, and achieved curricula in mathematics instruction. More specifically they should be knowledgeable about the interconnections among learning goals, assessment and teaching. They should understand different forms and purposes of assessment, including mathematics teacher-made student assessments used to inform future instruction and school-, district-, or state-mandated testing used to evaluate programs. Doctoral graduates should have opportunities to analyze and compare tests, including commercial achievement tests and state-constructed instruments. They should also know about national and international efforts to monitor student learning in mathematics and about the challenges associated with interpreting these results.

Part 3: Institutional Capacity Needed to Support Quality Doctoral Programs

Doctoral programs in mathematics education may be offered by a single institution or a collaboration of institutions. Regardless of the institutional structure, a high-quality doctoral program comprises more than a set of courses and a dissertation. Equally important is the environment created within an institution where students and faculty learn, work, and interact. It is critical that students have opportunities to work alongside and learn from active researchers and experienced collegiate teachers while they are engaged in their work. Doctoral programs in mathematics education should have resources of appropriate quality and sufficiency to support preparation of doctoral students and continual renewal of faculty. These institutional resources include the following:

1. A critical mass of faculty with expertise in mathematics education who provide program leadership and model professional behavior.
2. Faculty, possibly including some from outside mathematics education, who are engaged in research in mathematics education.
3. Adequate physical and technological facilities (e.g. computers, libraries, and meeting rooms) that support an active learning community of students and faculty.

4. Print and video resources (research journals in mathematics education; important reports; resources for teaching methods and content courses, including quality pre-K–14 mathematics curricula, methodology textbooks, state frameworks; and videos modeling mathematics teaching and learning) that facilitate professional growth.
5. Resources necessary to provide financial support for a critical mass of fulltime resident doctoral students.
6. Mentored internships focused on acquiring expertise in collegiate teaching, supervising student teachers, designing and implementing a research study, designing and facilitating professional development activities for teachers, preparing grant proposals, and writing papers for publication.
7. A supportive mathematics department that includes a group of mathematics faculty with interest in and a commitment to mathematics education.
8. Services available from all departments across the institution with appropriate expertise and a willingness to contribute to the program.
9. An environment that demonstrates respect for cultural, ethnic, racial, and individual diversity.

In closing, the Task Force recognizes that this report is not a definitive road map for doctoral programs in mathematics education. Neither complete nor perfect, it should be viewed as a work in progress. Although some in the mathematics education community may applaud this AMTE initiative and the report, others may be quick to critique both the effort and the product. To the former, we encourage you to send AMTE your ideas and suggestions for further refining the report. To the latter, we offer this challenge: If after a careful reading and analysis of the report, you believe a better blueprint exists that would be of greater use to the wide range of institutions with job opportunities that require a doctorate in mathematics education, please contact AMTE with your ideas for an alternate approach. Comments from all readers are welcome, as the resulting dialogue will serve to energize the mathematics education community and move the field forward.

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References

Glasgow, R. (2000). *An investigation of recent graduates of doctoral programs in mathematics education*. Unpublished doctoral dissertation, University of Missouri-Columbia.

- Hiebert, J., Kilpatrick, J., & Lindquist, M. (2001). Improving U. S. doctoral programs in mathematics education. In R. E. Reys & J. Kilpatrick (Eds.), *One field, many paths: U. S. doctoral programs in mathematics education* (pp. 153-162). Providence, RI: American Mathematical Society.
- National Research Council. (2000). *Summary report 2000: Doctoral recipients from United States universities*. Washington, DC: National Academy Press.
- Reys, R. E. (2000). Doctorates in mathematics education: An acute shortage. *Notices of the American Mathematical Society*, 47(10), 1267-1270.
- Reys, R. E. (2002). Mathematics education positions in higher education and their applicants: A many to one correspondence. *Notices of the American Mathematical Society*, 49(2), 202-207.
- Reys, R. E., & Kilpatrick, J. (Eds.). (2001). *One field, many paths: U. S. doctoral programs in mathematics education*. Providence, RI: American Mathematical Society