**Structures Lesson Plan**

**Objectives:**

The candidate will solve, analyze, and generate word problems involving various structures of addition and subtraction and multiple representations.

**Standards Alignment:**

|  |  |
| --- | --- |
| **Candidate Standard/Indicator** | **Tasks/ Activities and Nature of Alignment** |
| **EC.1. Deep Understanding of Mathematics:** Operations and Algebraic Thinking **(**Elaboration of C.1.1) | **Task 1** requires candidates to solve addition and subtraction problems using multiple representations and, later, analyze the structures of the problems they solved.  **Activity A** engages candidates in making connections between multiple representations of addition and subtraction problems and the structures those problems represent.  **Activity B** presents candidates with incorrect examples and representations of specific addition and subtraction structures. Candidates will develop deeper understanding of these structures through analyzing, discussing, and correcting these errors. |
| **EC.4. Tools, Tasks, and Talk as Essential Pedagogies for Meaningful Mathematics** (Elaboration of C.2.2 and C.2.3) | **Task 2**engages candidates in generating story problems of specific structures with unknowns in all positions. Candidates consider contexts, attending to student funds of knowledge and interests.  **Task 2** also offers candidates an opportunity to demonstrate representational competence, generate questions and prompts to support students’ problem solving, and anticipate student thinking. |
| **EC.7. Seeing Mathematics Through Children’s Eyes** (Elaboration of C.3.1) | After initially solving **Task 1**, analyzing the structures of the problems, and anticipating possible student solutions, candidates will examine a student’s solutions and video clips of explanations to these problems. Analysis of actual student work and reasoning will provide candidates an opportunity to consider the child’s understanding within a developmental progression and reveal how a child saw and solved the problems. |

**Materials:**

* Post-it notes (1 per student)
* Whiteboard/interactive whiteboard & markers
* Task 1 Worksheet
* Structures puzzle packets (1 per table or per small group)
* Student device (tablet or laptop)
* Projector/interactive whiteboard to display Task 2
* Error analysis handout (1 per student)
* Exit Ticket

**Context and Potential Adaptations for Other Contexts:**

The materials will be implemented across two elementary mathematics content and methods courses that comprise a math block. The instructors of these two courses will co-teach the lesson to one 150-minutes session with approximately 25 undergraduate elementary teacher candidates, seated in groups of 3 to 5.

[Adaptations for other course models](https://docs.google.com/document/d/1tTdc7fhCOMefNb4ZvvYesuTh2ckpKMXsT9h2qI1aiXY/edit), including separate methods and content courses, are included.

**Launch:** *(5 minutes)*

Candidates are given 2 post-it notes and asked to write a one-step addition or subtraction word problem with totals of less than 100.

Post-it notes are placed aside as the instructor identifies the lesson objectives. The instructor tells candidates that we will come back to these problems later in the lesson.

**Explore:**

[Task 1](https://drive.google.com/open?id=0B89fvskgjdqOYnRneWtTbGxKTjg): *Solving addition and subtraction word problems and comparing with a child’s solutions*

Candidates solve the six word problems on the Task 1 sheet, and are encouraged to include an equation and drawing/model with each of their solutions. (*5 minutes)*

After solving the problems, candidates work in partners or small groups to discuss the problems and alternative drawings, equations, and solutions they might anticipate from elementary students. *(5-10 minutes)*

Distribute a sample of a [child’s work on Task 1](https://drive.google.com/open?id=12uXPIHu3MHjAOMhDuGbMjtodS8tCN2mU) for partners/groups to analyze and discuss, then share out ideas to the whole group. How did the student’s work differ from theirs? What strategies did the student use and will they always be accurate? *(10-15 minutes)*

Watch videos of child solving Task 1 problems: [1](https://drive.google.com/open?id=0B89fvskgjdqOSHAtQTkyQnhxQzA) [2](https://drive.google.com/open?id=0B89fvskgjdqOcVhFVFpoYXVJT00) [3](https://drive.google.com/open?id=0B89fvskgjdqOaG9WOTV3RlNlblU) [4](https://drive.google.com/open?id=0B89fvskgjdqOUEFrbjVZOC1yVWM) [5](https://drive.google.com/open?id=0B89fvskgjdqOWFVlNlF6ZGxuN1E) [6](https://drive.google.com/open?id=0B89fvskgjdqONW5ROS1yV1hsaGs) and analyze how candidates’ analysis compared with the child’s explanations. You may wish to watch as many videos as time allows. Video 6 is the only example where the student begins with a subtraction equation (and the only take-away problem) In most cases, the child writes equations in the order the variables are described in the context, often solving what students wrote as subtraction equations by setting up an addition problem with a missing addend. The videos should also help candidates make sense of the child’s representations. *(15 minutes)*

[Sample of candidate work for Task 1](https://drive.google.com/open?id=1SzhJ6G_RyAZVEOR1IWEISyIMd4HAxSf8)

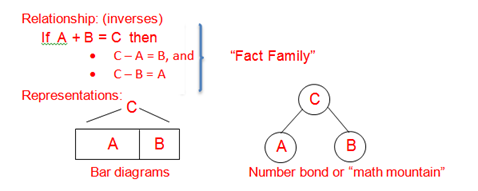
[Activity A](https://drive.google.com/open?id=1lBSyPdleZuLYljsnkU6yyU0BBOSgIUj6): *Puzzle with structures of addition and subtraction word problems.*

Mini-Lecture and Puzzle *(25 minutes)*

Introduce Activity A with a [mini-lecture](https://drive.google.com/open?id=1dDdDNTk62uyapU_KrbeWPiGYkjg2TVXW) to discuss [vocabulary](https://drive.google.com/open?id=1HoNaar6ij6Z4IslNCQ4UPLk7R0aS-T_M) related to [addition subtraction](https://drive.google.com/open?id=1KYdehhmXHdU8iGo3Wpj-0eIhE6EZ_U2b) problems, fact families highlighting the inverse relationship between addition and subtraction, and representations of fact families (including bar models and number bond models, see structures puzzle materials for examples).

*A + B vocabulary (addends, summands, terms, sum)*

*A – B vocabulary (difference, terms; optional: A = minuend, B = subtrahend; think “subtract the subtrahend”)*



Introduce the puzzle by explaining that addition and subtraction problems can be categorized into different structure types and that candidates will be assembling a puzzle to learn about these structures. Using one of the puzzle packets, demonstrate how the puzzle templates and puzzle pieces work. Encourage candidates to first match the structure titles to the appropriate template, then proceed to match the remaining pieces. Student work images: [1](https://drive.google.com/open?id=1EK5O8wDLYlUoZIw4oE2OkINPK2OiHSiy) [2](https://drive.google.com/open?id=16vv2aSeGlFjX5Sny8c24vef6QxFLt7cW)

Distribute puzzle packets to candidates and have them begin assembly. Puzzle packets include:

* + [Puzzle templates](https://drive.google.com/open?id=1wddNzht9eg87LYxOPE2QTSvo_BNlUdCZ)
  + Pre-cut [puzzle pieces](https://drive.google.com/open?id=1gEYmw_TNsaMvzz25iAXo-ZsOWb8HludV)
  + List of group [discussion questions](https://drive.google.com/open?id=1lBSyPdleZuLYljsnkU6yyU0BBOSgIUj6)

As candidates near completion of all four puzzles, display the [answer key](https://drive.google.com/open?id=1ZSCmm_aar5qBuujwy083Mztr_6nXav9w) so they can check their work. Then instruct candidates to respond to the discussion questions included in their puzzle packets.

Whole Class Discussion *(10-15 minutes)*

When all groups have completed the puzzles and had a few minutes do discuss the questions, engage the group in a whole class discussion highlighting key questions from the list.

*Have a structures resource page ready to display puzzle solutions during this discussion (and be prepared for a way to display/discuss errors and misconceptions).*

Which pieces were more difficult and why?

*Responses will vary. Discussions of these difficulties is intended to support productive struggle and deepen overall understanding of addition and subtraction structures.*

What is the difference between “add to/take away” problems and “put-together/take apart” problems?

*Some differences: “Add to/take away” problems typically involve the action of adding or subtracting the same item. E.g., I have some of an item and get more of/take away that same item.*

*“Put-together/take apart” problems are typically collections of different items. E.g., I have some of one type of item and some of another type of item, which make one collection comprised of the two types of items.*

What do “situation” and “solution equation” mean and why are these sometimes different?

*The logical way to represent a situation may not be how one would go about solving the problem. For instance, Jo had some stickers in her collection and got 3 more stickers at the dentist office. Now she has 11 stickers. How many stickers did she have before she went to the dentist? This problem can be solved as*

*11 - 3 = \_\_, but since the start is unknown, a more logical situation equation might be \_\_+3=11*

How do you decide which stories are compare problems?

*This distinction is likely to be fairly obvious to teachers. Focus discussion on how to support students in identifying and solving compare problems, including the use of bar diagrams, pictorial representations, and equations.*

Why are the models for the Compare problems different? (What are the ovals for?)

*In a bar diagram the large and the small bars represent a number of objects (in this case, buttons). The oval does not represent a quantity of buttons, but the difference between the two quantities.*

How could you represent some of these problems using number lines?

*The materials for the puzzle include only bar diagrams and math mountains, but students also should know how to model the situations using number lines.*

How might young children model the different types of problems with concrete objects?

*Children may act out the situation with physical objects using strategies such as counting on or counting down. This will be more straight-forward for those problems in which the initial quantity is known. For compare problems and problems in which the initial quantity is unknown, children may choose to build the two known quantities and line them up in a way that allows them to solve the problem.*

Why might it be important for teachers to know these types?

*Research indicates that students do not struggle equally with these types of problems. Comparison problems, in particular, can be challenging because drawing a pictorial representation is less intuitive for many students. Equipping students with a variety of representations and strategies can help support students’ development of conceptual understanding for addition and subtraction problem types.*

*Knowing about different types of problems can help teachers diagnose student errors and difficulties. Teachers who are aware of the variety of structures can also help to ensure students have ample opportunities to learn many structures over time. (We often tend to think mostly of add to/take away word problems when generating them ourselves.)*

[Task 2](https://drive.google.com/open?id=1cNqWaHmWq1DK3QatCETizrMixWGxiZ4SbSSb9M-gEPs): *Generating word problems of various structures within familiar contexts*

Revisit Launch Activity *(10 minutes)*

Have a whiteboard prepared with the following headings written at the top: *add-to, take away, put together, take apart, comparison*; and along the left side of the board: *total unknown, change unknown, start unknown*. (Depending on the timing of the class and facilities available, the instructor may wish to prepare the board before class, while candidates are working on Activity 1, or during a break immediately prior to this activity).

Now that candidates have had an opportunity to become familiar with the various structures for addition and subtraction word problems, ask them to take the problems they wrote on post-it notes at the beginning of class and place them in the appropriate column and row on the whiteboard. Mention that the rows (*total unknown, change unknown, start unknown*) do not hold for comparison so not to worry about the row for these types of problems.

Typically, the overwhelming majority of problems are *add to, take away*, or *put together,* and most are *total unknown*, as shown in this [instance](https://drive.google.com/open?id=1-z-EYDZT5KrtHhLePBWPq-olQS9Kq6cl). Discussion questions:

* What do you notice about the word problems our class generated?

There are hardly any comparison problems. Most of the problems are Total unknown. etc.

* Why might this matter in our teaching of addition and subtraction to students?

Students should have the opportunity to learn and solve problems involving a variety of structures.

When we select, modify, or generate problems ourselves, we should be mindful of the different structures of those problems.

Task 2 *(20-25 minutes)*

Use this as a way to introduce [Task 2](https://drive.google.com/open?id=1cNqWaHmWq1DK3QatCETizrMixWGxiZ4SbSSb9M-gEPs) in which candidates choose a context and generate specific types of addition and subtraction word problems within that context. The class will work in 12 pairs/small groups for this activity (shoulder partners, self-selected, or grouped according to instructor selection or strategy). A link to a Google Doc (or other shared working space) should be shared with candidates. This may be done with a QR code, directly emailing the link, posting within a course management system, etc.

Once groups are assigned and a number is given to each group, candidates should find their group number in the table on page 1 of [Task 2](https://drive.google.com/open?id=1cNqWaHmWq1DK3QatCETizrMixWGxiZ4SbSSb9M-gEPs), decide on a context that is familiar to everyone in the group (book, television show, movie, etc.). After identifying their contexts, groups should scroll to the next page and generate word problems within that context where they see their group number (2 problems per group). Problems should be directly typed in to the Google Doc so that they are visible to the instructor and peers. *(Instructor note: Many candidates find this activity rather enjoyable and may choose problems and contexts that wouldn’t necessarily be appropriate for students. The goal is for candidates to learn to generate problems in contexts and not to share these actual problems with students.)*

After all groups have finished, the instructor displays the completed task with the projector/interactive whiteboard. Candidates are asked to read through the problems on the table and identify any they have questions about or think may be inaccurate.

[Sample of candidate work for Task 2](https://drive.google.com/open?id=1p2688fOEZsFKRZ7U1K2sDh2Z2ThV11fZLKUKD536geo) (blinded)

Some points of discussion in the sample work above:

* Add to/join--start unknown problem 2: Spongebob has some crabby patties, Patrick has 5. All together they have 14 crabby patties. How many crabby patties does Spongebob have?

*This problem combines one quantity of patties with someone else’s quantity of patties. This is a better example of “put together/take apart” rather than “add to/join”*

* Compare--larger unknown problem 1: Meredith has been with Derek for 1 year. Allison has been with Derek for 4 more years. How many years in total has Derek been dating both of them?

*The beginning of this problem sets up a comparison situation. However, the question that is posed turns this into a two-step problem. One would have to first solve how long Derek has been with Allison (5 years), and then add 1 year + 5 years. Also, this context implies that they weren’t dating at the same time which may or may not be accurate.*

After this opportunity to generate word problems and analyze their own work, the next activity focuses more specifically on analyzing errors in word problem structures.

[Activity B](https://drive.google.com/open?id=1ZD0UGS2GNoXbJCydB7hq6d5sXMkbJrBK): *Analyzing errors in word problem structures.*

Error Analysis Handout *(10-15 minutes)*

Distribute the [Structure Lesson Error Analysis](https://drive.google.com/open?id=1ZD0UGS2GNoXbJCydB7hq6d5sXMkbJrBK) handout.

*Explain that the handout lists erroneous story problems based on written in response to Questions A-C (in bold font) by preservice teachers in a math for elem class. Explain that on the worksheet students only need to identify what is wrong with the sample questions provided (they do not need to rewrite the problem).*

Demonstrate the Stand-Up-Hand-Up-Pair-Up (Kagan) cooperative learning group strategy.

***Stand up*** *and look about the room to make eye contact with a first partner. Walk toward your partner and do a “high five” (****Hand up****) when you meet.* ***Pair up*** *with this partner to answer #1 together. When done, thank your partner and hold up two fingers – to show you are ready for #2 – and look around to find another partner. Work #2 with this new partner, thank him or her, then hold up three fingers and look for another partner ready to work #3. Continue in this way until all problems are complete. Early finishers may complete the last problem.*

Whole Class Discussion *(10 minutes)*

Review answers to the problems using the [answer key.](https://drive.google.com/open?id=1uY_okb6y0ApJW83XPYiVfEuLg91UVICK)

Then discuss using questions such as: Which errors were the most difficult to identify and why? The response to #2 has the right number sentence - why is it incorrect? The response in #8 is solved by addition as required - why is it incorrect? Will anyone share their story problem for the last question?

**Summarize:** *(10 minutes)*

Ask students to review the problems they solved for Task 1 and determine the structure for each of the 6 problems. Briefly talk to a partner or at tables about what you knew about these problems at the beginning of class, and what you know now. How are you better equipped to support students in learning this content?

Briefly share out contributions to the whole group before distributing [Exit Ticket](https://drive.google.com/open?id=1XiL_wPyL-XWr-2et0Yen1kJnRvs6oXlkDJvM6rHIwDs) to all students.

[Sample of candidate exit tickets](https://drive.google.com/open?id=1Pd8xPuOhwhCFkID45p55yn3ILzS2TX1C)

**Rubric**

A [rubric](https://drive.google.com/open?id=1yXBL7bYefBv61RvBOH54eCUNhY3tpm2ET6FVSky-D70) was designed to assist with interpreting exit slips or student work on Task 1.

You may wish to use the rubric on one or two problems from Task 1 and compare results with Exit slip.

To use, circle all that apply. This is not intended to assign a numerical score to student responses, but rather to support the instructor with interpreting student work. An [example](https://docs.google.com/document/d/1IizYE2OS1KaFEGf5mteJZ25QIIOOVv8L322sLgb8FLQ/edit?usp=sharing) of an evaluated exit ticket is offered with comments.

This rubric was applied to analyze work from 13 candidates for whom we had Task 1 and Exit tickets. The results of this analysis are described [here](https://drive.google.com/open?id=13n0U5DosPdUvMwwEp0BcA_VpF7hpSoUMMMRqi_Y4kME).