

AMTE Standards for Mathematics Teacher Preparation

Draft Version for Review, October 15, 2016

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Association of Mathematics Teacher Educators

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PREFACE

The future mathematical success of our nation's children is largely dependent on the teachers of mathematics they encounter from prekindergarten to grade 12 (PK-12). According to Tatto and Senk (2011), "If the quality of education for every child is to be improved, the education of teachers needs to be taken seriously" (p. 134). Teacher preparation programs in mathematics must ensure that all their candidates have the knowledge, skills, and dispositions so that all students have access and opportunities to have meaningful experiences with mathematics.

The Association of Mathematics Teacher Educators (AMTE) is the largest professional organization devoted to the improvement of mathematics teacher education. AMTE includes more than 1,000 members supporting preservice teacher education and professional development of teachers of mathematics at all levels from PK-12. AMTE members include professors, researchers, teacher leaders, school-based and district mathematics supervisors and coordinators, policymakers experts, graduate students, and others. The Standards described in this document reflect AMTE's leadership in shaping the preparation of PK-12 teachers of mathematics, including clearly articulated expectations for what *well-prepared beginning mathematics teachers* should know and be able to do upon completion of a certification program and the characteristics such programs must have to support their development.

Although what we know about the initial preparation of mathematics teachers has been limited, we have a growing research base that informs what teaching practices impact student learning and student experiences in mathematics classrooms. For example, research indicates that a focus only on teachers' behaviors has a less positive effect than a focus on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject (Carpenter, Fennema, Peterson, & Carey, 1988; Kennedy, 1998; Kwong et al., 2007; Philipp et al., 2007).

A number of recent documents address various aspects of the initial preparation of mathematics teachers.¹ Figure Preface.1 summarizes their focus. Although all of these standards inform mathematics teacher preparation, there is no single, comprehensive document addressing the initial preparation of mathematics teachers across PK-12. It is AMTE's intention that the standards provided in this document provide a clear, comprehensive vision for initial preparation of mathematics teachers, building on the standards briefly discussed above, and expanding on what beginning teachers of mathematics must know and be able to do, as well as the dispositions they must have, in order to increase equity, access, and opportunities for the mathematical success of all students. Given the challenges that teachers of mathematics face in preparing their students for future success, it is imperative that mathematics teacher educators are guided by a well-articulated vision to help prepare teachers of mathematics to meet those challenges. This document takes up that charge.

¹ For the purposes of this document, mathematics teacher preparation includes preparation to teach statistics, following common practice. However, we recognize that statistics and statistics education, while related to mathematics and mathematics education, are distinct.

PURPOSE

This document presents a set of comprehensive standards describing a national vision for the initial preparation of all teachers PK-12 who teach mathematics. That is, in addition to early childhood and elementary teachers who teach all disciplines, middle grade teachers, and secondary mathematics teachers, these standards are also directed towards special education teachers, teachers of emergent multilinguals, and all others who will have responsibility for aspects of student learning in mathematics.

These standards are intended to guide the improvement of individual teacher preparation programs, inform the accreditation process, and promote national dialogue and action related to mathematics teacher preparation. These standards are aspirational, advocating for mathematics teacher preparation practices that support candidates in becoming high quality teachers who are ethical advocates for children and effectively guide student learning in alignment with research and best practices, rather than describing minimum levels of competency needed by beginning teachers. The standards are intended to both build on existing research about mathematics teacher preparation (and existing standards) and to motivate research in areas that are less well understood.

AUDIENCE

The audience for these standards includes all those involved in mathematics teacher preparation, including faculty and others involved in the initial preparation of mathematics teachers; classroom teachers and other PK-12 school personnel who support student teachers and field placements; coordinators of mathematics teacher preparation programs; deans, provosts, and other program administrators who make decisions regarding content and funding of mathematics teacher preparation programs; CAEP, the largest accreditor of teacher education programs in the United States; state licensure or credentialing agencies/organizations; NCTM, the professional association responsible for setting standards for educator-preparation programs for preservice middle and high school mathematics; and other organizations, including specialized professional associations (e.g. NAEYC, CEC) and agencies focused on and involved in the preparation of mathematics teachers.

Table Preface.1. Documents on the Initial Preparation of Mathematics Teachers

Mathematical Education of Teachers II (METII) and Statistical Education of Teachers (SET)

The METII (Conference Board of Mathematical Sciences, 2012) addresses the mathematical content knowledge “well-started beginning teachers” should possess, and the SET (American Statistical Association, 2015) addresses the statistical content knowledge that preservice teachers should learn.

National Council of Teachers of Mathematics’ CAEP Standards

The NCTM CAEP Standards (NCTM, 2012a, NCTM, 2012b) describe what effective preservice teachers of secondary mathematics should know and be able to do, informing program reviews for middle and secondary mathematics programs.

Additionally, organizations not specific to mathematics address mathematical teaching and learning:

Council for Exceptional Children (CEC)

The CEC (2012) standards for beginning teachers requires that beginning professionals understand and use mathematics concepts in order to individualize learning for students.

National Association for the Education of Young Children (NAEYC)

The NAEYC (2010) professional standards describe the importance of knowing mathematics and teaching it in ways that promotes sense-making and nurtures positive development.

Council for the Accreditation of Educator Preparation (CAEP)

The CAEP further elaborates on content expectations for preservice teachers, describing knowledge, skills, and dispositions of effective teachers (CAEP, 2015).

Teacher Education and Development Study in Mathematics (TEDS-M)

The TEDS-M examined and discussed findings and challenges related to the mathematics education of future primary and secondary teachers (Tatto & Senk, 2011).

Finally, standards for experienced teachers that influenced these standards on the initial preparation of mathematics teachers:

InTASC Model Core Teaching Standards

The InTASC Model Core Teaching Standards (Council of Chief State School Officers, 2013) are used by states, school districts, professional organizations, and teacher education programs to support teachers.

National Board for Professional Teaching Standards (NBPTS)

The NBPTS were developed to provide certification designed to retain and recognize accomplished teachers and include certifications for early and middle childhood generalist (early childhood and elementary), and early adolescence (middle school), and adolescence and young adulthood (high school) mathematics teachers.

Association for Mathematics Teacher Education's (AMTE) Standards for Elementary Mathematics Specialists

AMTE's Standards for Elementary Mathematics Specialists (2013) outlines "particular knowledge, skills, and dispositions" needed by elementary mathematics specialists who "teach and support others who teach mathematics at the elementary level" (p. iv).

CHAPTER 1: INTRODUCTION

As a community, mathematics teacher educators have begun to define, research, and refine the characteristics of effective teachers of mathematics, and in particular the professional proficiencies of a well-prepared beginning teacher of mathematics. This document describes a set of proficiencies for well-prepared beginners and for programs preparing mathematics teachers. Although these proficiencies are grounded in the available research, that research is often insufficient to describe in detail the knowledge, skills or dispositions that will enable a beginner to be highly effective in their first years of teaching. Hence, the standards presented in this document are intended to engage the mathematics teacher education community in continued research and discussion about what candidates must learn during their initial preparation.

ASSUMPTIONS ABOUT MATHEMATICS TEACHER PREPARATION

The Standards for Mathematics Teacher Preparation were developed based on five foundational assumptions about mathematics teaching. These assumptions reflect the emerging consensus of those involved in mathematics teacher preparation in response to the needs of both their teacher candidates and the students those candidates will teach and underlie the standards presented in Chapters 2 and 3, as well as the grade-band elaborations in Chapters 4 through 7.

ASSUMPTION #1: ENSURING THE SUCCESS OF EVERY LEARNER DEMANDS A DEEP INTEGRATED FOCUS ON EQUITY IN EVERY PROGRAM THAT PREPARES MATHEMATICS TEACHERS.

Over the past decades, the need for a central focus on issues related to equity in mathematics education has become clear in reflecting on the uneven performance of students by various demographic factors (AMTE, 2015; NCTM, 2000, 2014a 2014b). Although equity, diversity, and social justice issues need to be specifically addressed as standards, they must also be embedded within all the standards that are described. Addressing these issues solely within the context of an “equity standard” might inadvertently imply that these issues are not important to the other standards; conversely, if they are not directly addressed, their centrality to the mission of mathematics teacher preparation can get lost. Thus, we believe that equity must both be addressed in its own right and embedded within every standard. Every standard must be built on the premise that it applies to all learners, recognizing that equity requires acknowledging the particular context and the needs and capabilities of each learner rather than providing identical opportunities to all students.

ASSUMPTION #2: TEACHING MATHEMATICS EFFECTIVELY REQUIRES CAREER-LONG LEARNING ABOUT TEACHING MATHEMATICS.

Experienced teachers reflecting on their first year of teaching mathematics frequently describe how much more they can now accomplish given their current level of teaching competence and understanding of the mathematics and students they are teaching. Teachers improve through reflective experience and through intentional efforts to seek additional knowledge. They use that knowledge to build their understanding of the mathematics they teach and to support their improvement in supporting students’ learning of mathematics. This process must begin during their initial preparation with clear expectations for this transition point, and then continue throughout their careers. Knowing that teaches candidates will complete teacher preparation programs without the expertise they will later develop focuses attention on priorities for teachers right from the start. Those become the knowledge, skills, and dispositions of a well-prepared beginner.

ASSUMPTION #3: LEARNING TO TEACH MATHEMATICS REQUIRES A CENTRAL FOCUS ON MATHEMATICS.

Many times teaching is approached as a general craft that is independent of the content being taught. However, effective mathematics teaching requires not just general pedagogical skills, but also content-specific knowledge, skills, and dispositions. In order to support student learning and develop positive dispositions toward mathematics, mathematics teachers, at every level of instruction, need deep and flexible knowledge of the mathematics they teach, how students think about and learn mathematics, instructional approaches that support mathematical learning, and the societal context in which mathematics is taught to effectively support student learning of and positive dispositions toward mathematics.

ASSUMPTION #4: MULTIPLE STAKEHOLDERS SHOULD BE RESPONSIBLE FOR AND INVESTED IN PREPARING TEACHERS OF MATHEMATICS.

Preparing teacher candidates to teach in ways that ensure all students learn important mathematics requires the concerted effort of everyone who holds a stake in students' future success. Mathematics teacher educators, mathematicians, general teacher educators, school administrators, classroom teachers, special education teachers, families and communities, policy-makers, and others in the educational system each play critical roles. When these groups send beginning mathematics teachers mixed messages about how mathematics is best taught and learned, those beginning teachers might come to hold an incomplete and fragmented vision of how to enact an effective mathematics learning environment for their students. Successful mathematics teacher preparation requires a shared vision of mathematics learning outcomes for students, of effective mathematics learning environments, and of the kind of experiences that best support a mathematics teacher's continuing growth and development. Moreover, stakeholders must both feel included in the development of that vision and accountable for enacting that vision.

ASSUMPTION #5: THOSE INVOLVED IN MATHEMATICS TEACHER PREPARATION MUST BE COMMITTED TO IMPROVING THEIR EFFECTIVENESS IN PREPARING FUTURE MATHEMATICS TEACHERS.

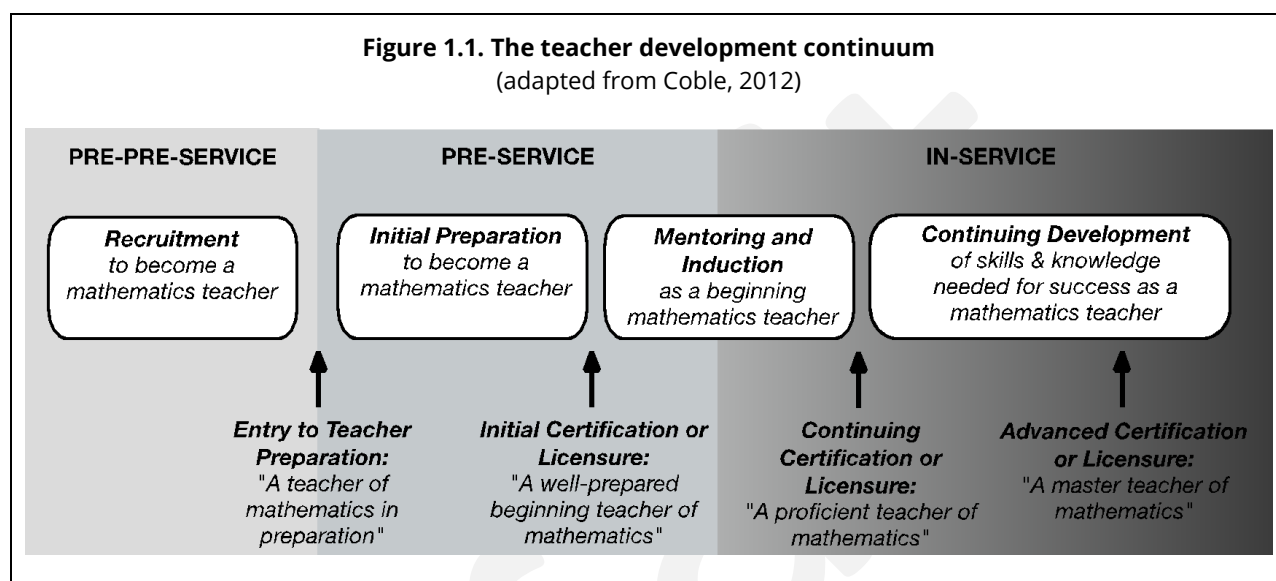
Mathematics teacher preparation programs must reflect research related to mathematics teacher preparation. The knowledge base for effective mathematics teacher preparation is far from complete. It is often not clear how the research base that does exist might apply across the range of contexts in which mathematics teacher preparation occurs; in the United States there are hundreds of institutions, as well as online and school district programs, where a person can become a teacher of mathematics and none of them are exactly alike. Program structures differ widely and the needs and backgrounds of their candidates vary. Thus, programs need to consider how existing research might apply to their context and how they can respond to issues not yet addressed by research. An emphasis on evaluating practices based on evidence will help to ensure effective decision-making. And as appropriate, the knowledge that is generated by particular programs should be contributed to the broader community of those involved in mathematics teacher preparation through publications, presentations at conferences, and other venues. Those able to conduct more formal research play an important role in exploring new directions for inquiry in mathematics teacher preparation.

CONCLUSION.

The standards in the chapters that follow provide clear expectations based on the current knowledge base and national recommendations related to preparing effective teachers of mathematics and provide a framework on which individual programs can study their practices, and on which researchers can begin to look across programs to better understand and investigate critical aspects in the preparation of a well-prepared beginning teacher of mathematics.

TEACHER PROFESSIONAL CONTINUUM

As previously stated in Assumption 2, the development of teachers' content and teaching knowledge, skills, and dispositions develops over a career-long trajectory, as depicted in Figure 1.1. For example, the Interstate Teacher Assessment and Support Consortium (InTASC) developed learning progressions to describe "a coherent continuum of expectations for teachers from beginning through accomplished practice" (Council of Chief State School Officers, 2013, p. 6).



The standards in this document primarily address the initial two phases of the trajectory depicted in Figure 1.1, from recruitment of teacher candidates into a teacher preparation program to their entry into the profession. These standards thus set a strong foundation for the continuing growth and development of teachers of mathematics across the continuum.

A WELL-PREPARED BEGINNING TEACHER OF MATHEMATICS

The standards in this document describe targets for what a well-prepared beginning teacher of mathematics should know and be able to do, as well as productive dispositions they should develop. Although not every candidate will be able to achieve every target, well-prepared beginning mathematics teachers should be committed to support the mathematical success of all students, and with proper support and incentives, they will continue to become more effective over the course of their careers.

This chapter described the overall framework including a set of assumptions for the document. Chapter 2 provides standards for the professional knowledge, skills, and dispositions that well-prepared beginning mathematics teachers should possess related to content, teaching, learners and learning, and the social context of mathematics education.

Chapter 3 describes standards for mathematics teacher preparation programs designed to develop the knowledge, skills, and dispositions of their teacher candidates described in Chapter 2.

Chapters 4 through 7 make specific recommendations about the preparation of mathematics teachers at different levels of instruction or grade bands, separated due to the differential needs for teachers of learners in

different ages and to align them to previous standards (such as NCTM's *Principles and Standards for School Mathematics*). These include Prekindergarten to grade 2, grades 3 through 5, grades 6 through 8, and grades 9 through 12.

Chapter 8 provides attention to assessment of candidates and programs.

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CHAPTER 2. CANDIDATE KNOWLEDGE, SKILLS, AND DISPOSITIONS

Teaching is a complex enterprise, and teaching mathematics is particularly complex. Thus, it is not surprising that initial preparation that focuses on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject is more effective than preparation that focuses on teachers' specific behaviors (cf., Ball & Forzani, 2011; Philipp et al., 2007). As described in *Accreditation Standards and Evidence: Aspirations for Educator Preparation* (Council for the Accreditation of Educator Preparation (CAEP), 2013), teacher candidates must learn “critical concepts and principles of their discipline and, by completion, are able to use discipline-specific practices flexibly” (p. 3). This chapter describes the specific knowledge, skills, and dispositions that well-prepared mathematics teacher candidates will know and be able to do upon completion of an initial preparation program. We use the phrase “well-prepared beginning teachers of mathematics” (“well-prepared beginners” for short), referring to those who are starting their careers after completion of a teacher preparation program.

Organization of this Chapter

This chapter includes four equally important and interrelated standards that describe the knowledge, skills, and dispositions that well-prepared beginners should acquire. The first standard, “Knowledge of Mathematics for Teaching,” describes the content knowledge involved in the teaching of mathematics. The second, “Knowledge and Practices for Teaching Mathematics,” describes research-based practices or strategies for effective mathematics teaching. The third, “Knowledge of Students as Learners of Mathematics,” describes what teachers should know about their students’ mathematical knowledge, skills, representations, and dispositions, for both individual students and groups of students. The final standard in this chapter, “Social Contexts of Mathematics Teaching and Learning,” describes the knowledge and dispositions beginning teachers should have about the social, historical, and institutional contexts of mathematics that impact teaching and learning, themes that are also woven into the first three standards.

As indicated in Table 2.1, each standard includes specific indicators, along with accompanying explanations. These standards and indicators apply to all well-prepared beginning teachers of mathematics from prekindergarten through high school.

Table 2.1 Standards for Well-prepared Beginning Teachers of Mathematics and Related Indicators

Standard	Related Indicators
Standard C.1: Knowledge of Mathematics for Teaching <i>Well-prepared beginning teachers of mathematics possess appropriate mathematical knowledge of and skill in mathematics needed for teaching. They engage in appropriate mathematical practices and support their students in doing the same. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students' mathematical productions.</i>	C.1.1. Core Content Knowledge C.1.2. Mathematical Practices and Processes C.1.3. Mathematical Dispositions C.1.4. Analyze the Mathematical Content of Curriculum C.1.5. Mathematical Knowledge to Analyze Mathematical Thinking C.1.6. Use Mathematical Tools and Technology
Standard C.2: Knowledge and Pedagogical Practices for Teaching Mathematics <i>Well-prepared beginning teachers of mathematics have a foundation of pedagogical content knowledge, effective and equitable mathematics teaching practices, and a positive and productive disposition toward teaching mathematics to support students' sense-making, understanding, and reasoning.</i>	C.2.1. Promote Equitable Teaching C.2.2. Plan for Effective Instruction C.2.3. Implement Effective Instruction C.2.4. Analyze Teaching Practice C.2.5. Enhance Teaching through Collaboration with Colleagues, Families, and Community Members
Standard C.3: Knowledge of Students as Learners of Mathematics <i>Well-prepared beginning teachers of mathematics have foundational understandings of students' mathematical knowledge, skills, and dispositions. They also know how these understandings can contribute to effective teaching and are committed to expand and deepen their knowledge of students' as learners of mathematics.</i>	C.3.1. Students' Thinking about Mathematics Content C.3.2. Students' Engagement in Mathematical Practices C.3.3. Students' Mathematical Dispositions
Standard C.4: Social Contexts of Mathematics Teaching and Learning <i>Realizing that the social, historical, and institutional contexts of mathematics impact teaching and learning, well-prepared beginning teachers are knowledgeable about and committed to their critical role as advocates for every mathematics student.</i>	C.4.1. Access and Advancement C.4.2. Mathematical Identities C.4.3. Students' Mathematical Strengths C.4.4. Power and Privilege in the History of Mathematics Education C.4.5. Ethical Practice for Advocacy

What Should Well-Prepared Beginning Teachers of Mathematics Know and Be Able to Do, and What Dispositions Should They Develop?

The guiding question for this chapter is, “Recognizing that learning to teach is an ongoing process over many years, what are reasonable expectations for the most important knowledge, skills, and dispositions that *beginning* teachers of mathematics must possess to be effective?” This is a difficult question to answer, as some aspects of teaching are not going to be well learned initially, even though they may be critically important to student learning. This is also a significant equity issue, as students with the greatest needs are often taught by teachers with the least experience (Kalogrides, Loeb, & Béteille, 2012; Oakes, 2008).

Mathematics teachers, from the very beginning of their careers, must understand the mathematical content knowledge for the age groups or grades that they may teach, along with content that comes before and after those age groups or grades—and in a different and deeper way than often presented in textbooks, curriculum documents, or standards. Such knowledge impacts their students’ learning (e.g., Hill, Rowan, & Ball, 2005; National Mathematics Advisory Panel, 2008).

Well-prepared beginners must be ready to teach every child in their first classrooms. Although pedagogical skills develop over time, beginners must have an initial repertoire of effective and equitable teaching strategies; for example, in selecting tasks, orchestrating classroom discussions, building on prior knowledge, and connecting conceptual understanding and procedural fluency (National Council of Teachers of Mathematics (NCTM), 2014). All teachers, including well-prepared beginners, must hold positive dispositions about mathematics and mathematics learning, such as the notions that mathematics can and must be understood, and that all students can develop mathematical proficiency, along with a commitment to imbue their students with similar beliefs and dispositions.

Being able to teach effectively requires knowledge of learners and learning, both general pedagogical knowledge and knowledge specific to the learning and teaching of mathematics. Knowing learners includes knowing about their background, interests, strengths, and personalities, as well as knowing how students *think* and *learn* related to the mathematics they will be teaching, including possible misconceptions and creative pathways they may take in learning (Ball & Forzani, 2011; Clements & Sarama, 2014; Sztajn, Confrey, Wilson, & Edgington, 2012). Well-prepared beginners must understand—at least at an initial level—how to assess the understandings and competencies of their students and use this knowledge to plan and modify instruction using research-based instructional strategies (e.g., Ball & Forzani, 2011; Shulman, 1986).

Mathematics teaching and learning are influenced by social, historical, and institutional contexts. Beginning teachers must be aware of learners’ social, cultural, and linguistic resources; know learners’ histories; and recognize how power relationships affect students’ mathematical identities, access, and advancement in mathematics (e.g., Gutiérrez, 2013; Martin, 2015; Strutchens et al., 2012; Wager, 2012). For example, classroom dynamics and social interactions strongly influence students’ emerging mathematical identities, which in turn impacts the students’ learning opportunities. In short, well-prepared beginners must be ethical advocates for each of their students.

Standard C.1: Knowledge of Mathematics for Teaching

Well-prepared beginning teachers of mathematics possess appropriate mathematical knowledge of and skill in mathematics needed for teaching. They engage in appropriate mathematical practices and support their students in doing the same. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students’ mathematical productions.

Knowledge of Mathematics for Teaching extends beyond general subject matter knowledge. It includes the knowledge of mathematics that teachers need to design and adapt lesson plans, anticipate student questions and misconceptions, evaluate student work, use proper notation and vocabulary, and the knowledge of how mathematics content is connected across grade levels in order to convey mathematical ideas accurately, anticipating how these ideas develop over time. This knowledge is part of what Shulman (1986) described as Pedagogical Content Knowledge: “the ways of representing and

formulating the subject that make it comprehensible to others... an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons” (p. 9).

Indicator C.1.1. Core Content Knowledge: *Well-prepared beginning teachers of mathematics have a solid and flexible knowledge of core mathematical concepts and procedures they will teach, along with knowledge beyond what they will teach and mathematics foundational to those core mathematical concepts and procedures.*

Well-prepared beginners can understand and solve problems in more than one way, explain the meaning of key concepts, and explain the mathematical rationale underlying key procedures. For example, a well-prepared beginner for grades 3 through 5 should recognize that simplifying $3 \div \frac{1}{5}$ suggests the question, “How many fifths are in 3?” Using a visual diagram as in Figure 2.1, as well as considering that there are five fifths in one whole, leads to the realization that the answer will be 3×5 or 15.

Figure 2.1. Fraction bar representation of the problem $3 \div \frac{1}{5}$.



This result will be generalized to recognize that dividing by any unit fraction is equivalent to multiplying by the denominator. Thus, this procedure is built on a solid and flexible understanding of underlying mathematics. (See chapters 4 through 7 for additional examples of the specific content that should be addressed for well-prepared beginners at each grade-band.)

Indicator C.1.2. Mathematical Practices and Processes: *Well-prepared beginning teachers of mathematics have a solid and flexible knowledge of mathematical processes and practices, recognizing that these are tools that people use to solve problems and communicate ideas.*

The mathematical knowledge of well-prepared beginning teachers includes their ability to use mathematical processes and practices (CCSSM, 2010; NCTM, 2000) to solve problems. They use mathematical language with care and precision. They can explain their mathematical thinking using grade-appropriate concepts, procedures, and language, including knowing grade-appropriate definitions and interpretations for key mathematical concepts. They can apply their mathematical knowledge to real-world situations by using mathematical modeling to solve problems appropriate for the grade levels and the students they will teach. They are able to effectively use representations and technological tools

appropriate for the mathematics content they will teach. They regard doing mathematics as a sense-making activity that promotes perseverance, problem posing, and problem solving. In short, they exemplify the mathematical thinking that will be expected of their students.

Well-prepared beginning teachers of mathematics recognize processes and practices as they emerge in their mathematical thinking and highlight these actions and behaviors when they observe them in others. Over time, beginning teachers can 1) better distinguish intricacies among the various processes and practices, determining those that at the crux of a mathematical investigation; and 2) see the interrelationships among the processes and practices.

Well-prepared beginners understand that mathematics is a human endeavor that is practiced in and out of school, across many facets of life. They know that mathematics has a history, and includes contributions from people with different cultural, language, religion, gender, and race/ethnicity backgrounds. Much of mathematics is based on constructed conventions and agreements about the meaning of words and symbols. And, that there are also many ways that mathematics differ from country to country, as well as from individual to individual. They are aware that, for many procedures processes, what are described as standard algorithms, may not be standard everywhere (i.e., many standard algorithms in the U.S. are different from than algorithms in other countries), and, there are alternative algorithms, some of which have different, desirable properties that make them worth knowing. This idea is elaborated in later sections of this chapter, as well as chapters 4 - 7.

Indicator C.1.3. Mathematical Dispositions: *Well-prepared beginning teachers of mathematics understand that success in mathematics depends on a productive disposition towards the subject and hard work.*

Well-prepared beginning teachers of mathematics expect mathematics to be sensible, useful, and worthwhile for themselves and others, and believe that all people are capable of thinking mathematically and are able to solve sophisticated mathematical problems if they work at it (National Research Council [NRC], 2001). They believe that requisite characteristics of high-quality teaching of mathematics include a commitment to sense-making in mathematical thinking, teaching, and learning and to developing “habits of mind,” including curiosity, imagination, inventiveness, risk-taking, and persistence. For example, when faced with something that challenges common practice or their current understandings or beliefs, well-prepared beginners have the mathematical disposition to investigate the proposed idea, applying their own critical thinking, rather than resorting to an external authority.

Indicator C.1.4. Analyze the Mathematical Content of Curriculum: *Well-prepared beginning teachers of mathematics can read, analyze, and interpret curriculum, content trajectories, standards documents, and assessment frameworks for the grades in which they are being prepared to teach.*

The alignment of standards, curriculum, and assessment is critical in designing a cohesive, well-articulated curriculum. Well-prepared beginning teachers of mathematics are aware that the mathematics they teach is based on a variety of documents that are often nested within each other. They understand the complex connections among standards, curriculum documents, instructional

materials, and assessment frameworks. They have the content preparation and the disposition to analyze instructional resources, such as textbook lessons and assessments, and resources on the internet, to see if these resources fully address the content expectations described in standards and curriculum documents. When the materials fall short of the standards or expectations, well-prepared beginners are able to make decisions about whether to replace or adapt the materials to better address the content expectations.

Well-prepared beginners realize that in addition to the curriculum and standards that they are accountable to teach, other resources can support their efforts to design rigorous, coherent mathematics instruction, such as content or standards progressions (cf. <https://www.turnonccmath.net/> or <http://ime.math.arizona.edu/progressions>) that describe relationships among standards within and across grades. Note that there are other types of progressions that may need to be considered in addition to content progressions, such as developmental progressions or learning trajectories. They understand the content within these materials and can discuss them with families of their students, colleagues, and administrators in ways that make sense to the different audiences.

Through analyzing available resources, well-prepared beginners are able to make decisions about the sequencing and time required to teach the content in depth, as well as make important connections between the mathematics that comes in the grades or units before what they are teaching, and the units and grades after the one they are teaching. See chapters 4 through 7 for an elaboration of what well-prepared beginning teachers of mathematics need to know for their specific grade level.

Indicator C.1.5. Mathematical Knowledge to Analyze Mathematical Thinking: *Well-prepared beginning teachers of mathematics can analyze different approaches to mathematical work and respond appropriately.*

Well-prepared beginners can analyze both written and oral mathematical productions related to key mathematical ideas, and look for and identify sensible mathematical reasoning even when that reasoning may be atypical or different from their own. Well-prepared beginners value different approaches to solving a problem, recognizing that there is more to doing mathematics than finding an answer. The task in Figure 2.2, which might be used in a mathematics methods or mathematics content course for teachers, exemplifies the kind of mathematical analysis well-prepared beginners need to be able to do.

Figure 2.2. Sample task for prospective teachers.

One number is 3 times another number, and their sum is 30. What are the two numbers?

Four responses follow:

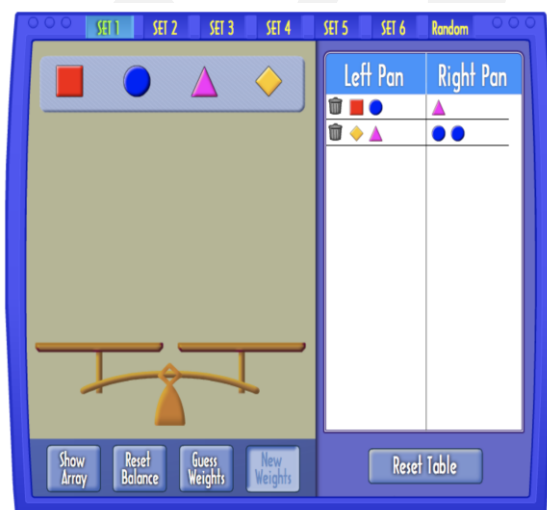
- 1) $30 \div 3 = 10$, so 10 and 30.
- 2) $30 \div 4 = 7.5$, so 7.5 and 22.5
- 3) Half of 30 is 15. Half of 15 is 7.5, so go up and down 7.5 from 15.
- 4) $x + 3x = 30$, so $x = 7.5$

What question would you ask to clarify each person's thinking?

Indicator C.1.6. Use Mathematical Tools and Technology: *A well-prepared beginning teacher of mathematics must be proficient with tools and technology designed to support mathematical reasoning and sense making, both in doing mathematics themselves and in supporting student learning of mathematics.*

Well-prepared beginning teachers of mathematics should be proficient in using both electronic and non-electronic tools such as concrete manipulatives as a tool for solving mathematical problems and as a means of enhancing or illuminating their thinking. In particular, use of technology is an expected part of society and the workforce and is an important tool for doing mathematics. Well-prepared beginners should be particularly prepared to use “mathematical action technologies” (cf. NCTM, 2014, p. 79), powerful tools for doing mathematics that will be a part of the lives of the students they teach. At the secondary level, such tools may include dynamic geometry environments, spreadsheets, computer algebra systems such as WolframAlpha, and statistical packages. They may also include apps that are specially designed to explore particular mathematical contexts, such as the app from NCTM’s Illuminations website pictured in figure 2.3, which simulates weighing objects on a pan balance to determine their relative weights. “Virtual manipulatives,” interactive electronic depictions of physical manipulatives, can support sophisticated explorations of mathematical concepts (Moyer, Niezgoda, & Stanley, 2005).

Figure 2.3. Pan balance app -- <http://illuminations.nctm.org/Activity.aspx?id=3531>



Well-prepared beginners should have the knowledge needed to “make sound decisions about when such tools enhance teaching and learning, recognizing both the insights to be gained and possible limitations of such tools” (NCTM, 2012a, p. 3). Not every tool, whether electronic or physical, is appropriate in every situation, and different tools may provide different insights into a context. Making a decision about what tool or tools to use to represent a context is itself a mathematical act.

Standard C.2: Knowledge and Pedagogical Practices for Teaching Mathematics

Well-prepared beginning teachers of mathematics have a foundation of pedagogical content knowledge, effective and equitable mathematics teaching practices, and a positive and productive disposition toward teaching mathematics to support students' sense-making, understanding, and reasoning.

Indicator C.2.1. Promote Equitable Teaching: *Well-prepared beginning teachers of mathematics structure learning opportunities and use teaching practices that provide access, support, and challenge in learning rigorous mathematics to advance the learning of every student.*

Teaching for access and equity is evidenced as well-prepared beginning teachers view their role as being able to develop robust and powerful mathematical identities in their students, demonstrating a commitment to view each student as a capable and unique learner of mathematics. Well-prepared beginning teachers embrace and build on students' current mathematical ideas and on students' ways of knowing and learning, including attention to students' culture, race/ethnicity, language, gender, socioeconomic status, cognitive and physical abilities, and personal interests. They also attend to developing students' identity and agency so that students can see mathematics as a component of their culture and see themselves in the mathematics. For instance, well-prepared beginners often connect mathematics to students' everyday experiences or ask students to tell stories, because they know that making mathematics relevant to students' lives can make the learning experience more meaningful and raise achievement (Turner, Celedón-Pattichis, Marshall, & Tennison, 2009). Ensuring equitable mathematics learning outcomes for all students is an essential goal and a significant challenge. Achieving this goal necessitates clear and coherent mathematical goals for students' learning, expectations for the collective work of students in the classroom, effective methods of supporting the learning of mathematics by all students, and providing students with appropriate tools and resources targeted to their specific needs.

Teaching with a commitment to access and equity means striving to reach each student whose life is impacted by what occurs in the mathematics classroom. Well-prepared beginning teachers plan for and use an "equity-based pedagogy" (AMTE, 2015) by structuring learning opportunities to provide access, support, and challenge in learning mathematics. This includes considering students' individual needs, cultural experiences, and interests, as well as prior mathematical knowledge, when selecting tasks and planning for mathematics instruction (Leonard, Brooks, Barnes-Johnson, & Berry, 2010). For example, what everyday, informal language might support or hinder the specialized use of language in mathematics? What are students' prior experiences with specific mathematical representations or strategies? What scaffolds are needed to support students with special needs? What books, movies, television shows, and video games are currently popular with students and could be incorporated into tasks or problem solving opportunities?

Well-prepared beginning teachers of mathematics strongly believe that all students can learn mathematics with understanding and take conscious and intentional actions to build students' agency as mathematical learners (AMTE, 2015; Gutiérrez, 2009). That is, they believe in each student's ability to make sense of mathematical tasks and situations, to engage in mathematical discourse, and to judge the

validity of solutions. For example, the beginning teacher envisions a classroom community in which students present ideas, challenge each other, and construct meaning together and where varied mathematical strengths are valued and celebrated. Well-prepared beginners understand the importance of communicating the relevance of mathematics, specifically, that students can use mathematics to address problems/concerns in their homes and communities.

Additionally, well-prepared beginners intentionally foster a growth mindset among students about learning mathematics and persistently counter manifestations of fixed mindsets (e.g., that some people are good at mathematics and others are not). This includes public praise for contributions, use of applicable strategies, and perseverance (Boaler, 2016; Dweck, 2008). For example, well-prepared beginners acknowledge mistakes as critical for learning and support students in viewing mistakes as an important part of the learning process.

Indicator C.2.2. Plan for Effective Instruction: *Well-prepared beginning teachers of mathematics attend to a multitude of factors, including content, student learning needs, task selection, and assessment, in designing mathematical learning opportunities for students.*

Careful and detailed planning is necessary for designing lessons that build on students' mathematical thinking while developing key mathematical ideas. Well-prepared beginners realize it will take substantial amounts of time to plan. They know it is important to have a clear understanding of the mathematics content and mathematics learning goals for each unit and lesson, as well as how these particular goals fit within a developmental progression of student learning (Daro, Mosher, & Corcoran, 2011). Well-prepared beginners are able to articulate and clarify mathematics learning goals during the planning process, knowing the goals are the starting point that "sets the stage for everything else" (Hiebert, Morris, Berk, & Jansen, 2007, p. 57).

Considering the prior knowledge and experiences that students bring to a lesson and how the task will be set up or launched so as to ensure that all students have access to the content and context, and that the content and context are meaningful to students is essential for effective instruction (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; NCTM, 2009). Therefore, well-prepared beginners strive to design classroom environments that provide students with the opportunities to communicate their thinking, listen to the thinking of others, connect mathematics to a variety of contexts, and make connections across mathematical ideas and different subject areas. They plan purposeful and meaningful questions that probe student thinking, make the mathematics visible for discussion, and encourage reflection and justification (NCTM, 2014).

Effective mathematics planning includes the selection of meaningful tasks to motivate student learning, develop new mathematical knowledge, and build connections between conceptual and procedural understanding. Well-prepared beginners understand that providing students opportunities to think, reason, and solve problems requires cognitively challenging mathematical tasks (Stein, Smith, Henningsen, & Silver, 2009). Additionally, students must engage on a regular basis with mathematical tasks that promote reasoning and problem solving, provide multiple entry points, have high ceilings to offer challenges, and support varied solution strategies (NCTM, 2014). Such tasks can only be identified by considering the ways in which students might solve the task. Therefore, well-prepared beginners analyze tasks and lessons, anticipating students' approaches and responses (Gravemeijer, 2004; Stigler

& Hiebert, 1999). They understand that anticipating students' responses involves considering the array of strategies, both conventional and unconventional, which students might use to solve the task, and how those strategies relate to the mathematical concepts, representations, and procedures that students are learning (Stein, Engle, Smith, & Hughes, 2008). Well-prepared beginners attend to the needs of their students in their planning of lessons and units. This means planning that incorporates inclusion and equity-based teaching practices.

Formative assessment is an integral aspect of effective instruction, therefore lesson planning must include a plan for monitoring and assessing student understanding (Black & Wiliam, 1998). Well-prepared beginners have a repertoire of strategies to elicit evidence of students' progress toward the intended mathematics learning goals, such as being able to use observation checklists, interviews, writing prompts, exit tickets, quizzes and tests. They realize that while they have anticipated student responses, the evidence from these assessments may require a departure from the planned lesson or may impact subsequent lessons within a unit of study.

Indicator C.2.3. Implement Effective Instruction: *Well-prepared beginning teachers of mathematics use a core set of pedagogical practices that are effective for developing meaningful student learning of mathematics.*

Teachers must not only understand the mathematics they are expected to teach (Ball, Thames, & Phelps, 2008) and understand how students learn mathematics (Fuson, Kalchman, & Bransford, 2005), they must be skilled in using content-focused instructional pedagogies that advance the mathematics learning of every student (Forzani, 2014). Well-prepared beginning teachers of mathematics have begun to develop skillful use of a core set of effective teaching practices, such as those described in *Principles to Actions* (NCTM, 2014) and listed in Figure 2.4 below.

Figure 2.4. Effective Mathematics Teaching Practices (NCTM, 2014, p. 10)

- **Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
- **Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
- **Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
- **Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
- **Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to

assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

- **Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
- **Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
- **Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Effective instruction includes establishing a classroom community centered on mathematical discourse and sense-making. Students are engaged with mathematical tasks that promote reasoning and problem solving, work toward clearly established mathematics learning goals for developing conceptual understanding and build toward procedural fluency. Accomplished teachers realize the importance of giving students time to struggle productively in exploring these mathematical tasks and allocate substantial instructional time for students to use, discuss, and develop understanding of important mathematical ideas. Experienced teachers are also adept at monitoring students as they work on tasks, asking purposeful questions to support student learning without lowering the cognitive demand or taking over the thinking and reasoning of students (Stein, Grover, & Henningsen, 1996; Stigler & Hiebert, 2004). Well-prepared beginning teachers of mathematics enter classrooms with commitment to and initial skills for enacting effective mathematics instruction. They are clear on the mathematics learning goals for lessons, how those goals relate to the selected tasks, and how to use the goals to guide their instructional decisions throughout a lesson. Furthermore, they discuss the mathematics learning goal with their students and how it relates to the current work and tasks, because this level of transparency helps students to focus and monitor their own learning throughout a lesson.

Accomplished teachers know their students well and are able to draw on students' prior knowledge and experiences, including culture, language, and interests, to ensure students are able to connect with the mathematical ideas and to build students' sense of identity and agency as mathematical learners (Turner et al., 2012). Students are positioned as authors of ideas, who discuss, explain, and justify their reasoning using varied representations and tools. In classrooms that support these teaching practices, students talk with, respond to, and question one another as they work with partners and in small groups, as well as in whole class discussions. Effective teachers also carefully monitor the mathematical learning of their students as they engage with tasks and with each other, selecting and sequencing student approaches for whole class discussion. While well-prepared beginners can see key mathematical ideas within students' representations, the accomplished teacher can weave a mathematical idea across many representations (e.g., proportional relationships shown with discrete objects, tables, tape diagrams, double number lines) in ways that help students see connections among representations and affordances of different representations. Skillful orchestration of student interactions and facilitation of meaningful mathematical discourse is complex and takes years for teachers to fully develop. Well-prepared beginners provide opportunities for student-to-student dialogue as they work together on

mathematical tasks with each other and as they engage in whole class discussions to share, compare, and analyze student strategies and arguments. Beginning teachers know the purpose of whole class discussions is not “show-and-tell” but rather an intentional discussion of selected and sequenced student approaches and use of mathematical representations and tools that move students through a trajectory of sophistication toward the intended mathematics learning goal of the lesson. In addition, they know that productive mathematical discourse does not just happen, but requires establishing and reinforcing norms for classroom discourse.

The increasing diversity of students in classrooms requires that experienced and beginning teachers alike commit themselves to eliciting and using student thinking to not only assess student progress in understanding the mathematics, but also to adjust instruction during the lesson in ways that support and extend learning. Well-prepared beginners check-in on student understanding and reasoning at multiple points throughout a lesson. This might include posing purposeful questions to gather information or probe student understanding as they work individually or in small groups, or asking students to respond in writing to an open-ended prompt on their whiteboards during a lesson or completing an exit ticket at the end of a lesson. In addition, well-prepared beginners consciously involve each student in the lesson, even keeping track of who participates in discussions. Accomplished teachers go further to notice patterns of participation and they are active in addressing patterns that are unproductive or exclusionary because they know that this presents barriers to students’ learning as well as limits access to students’ mathematical thinking and current levels of understanding.

Indicator C.2.4. Analyze Teaching Practice: *Well-prepared beginning teachers of mathematics are developing as reflective practitioners who elicit and use evidence of student learning and engagement to analyze their teaching.*

To effectively reap the benefits of the process of reflection, teachers must base their instructional decisions on evidence of student thinking and reasoning (William & Leahy, 2015) (See Standard 3.3). Well-prepared beginners analyze the formative assessments used in a lesson to determine student conceptions and determine future instruction. They recognize that their analysis must go beyond identifying an overall need, but determine precise issues that an intervention could directly support (Hodges, Rose, & Hicks, 2012). For example, a diagnostic interview using a missing addend problem could reveal a gap in the student’s knowledge about the meaning of the equal sign. This information can lead to changes in instruction, such as incorporating an actual balance, or ensuring that equations are written in a variety of ways in future lessons.

Well-prepared beginning teachers of mathematics recognize that the process of data collection, analysis, and reflection and the corresponding revision to classroom practices is a systematic and continuous process and grows in sophistication with teaching experience to be increasingly more embedded in instruction (rather than a post hoc task such as an exit ticket). Eventually this deliberate examination of practice, in tandem with collecting evidence of student thinking and performance, helps well-prepared beginning teachers become more objective about their own teaching practice. A variety of tools exist that allow teachers to design and analyze mathematics lessons. These tools are designed to gather evidence on students’ multiple mathematical knowledge bases and culturally responsive teaching (Aguirre & Zavala, 2013; Roth McDuffie et al., 2014).

Reflecting on one's teaching should not remain at the individual level. The continuous monitoring of one's practice leads well-prepared beginners to seek out collaborators or critical friends (Schuck & Russell, 2005) to observe each other's teaching, examine students' work samples as a team, and in concert brainstorm next instructional steps. It is often hard to abandon carefully designed instructional plans, but when others can point to better ways of proceeding, change is easier than sustaining teaching practices that are not working well for students' long-term gains. Through collaborative efforts and adjustments, the outcome is not that time is lost but instead that it is used more wisely to strengthen teaching practice. By asking important questions like, "How might I get better at this practice I am developing?" and "What other teaching practices might I prioritize?," well-prepared beginning teachers of mathematics are open to feedback and engage in the process of making conscious instructional choices based on actual evidence of teaching practice.

Professional norms are changing to encourage teacher-to-teacher collaboration through professional learning communities and formal mentoring and coaching programs. In addition, social media (e.g., blogs; Instagram; Mathematics twitter blogosphere (MTBoS)) are providing virtual spaces for teachers to connect and reflect on their instructional practice.

Indicator C.2.5. Enhance Teaching through Collaboration with Colleagues, Families, and Community Members: *Well-prepared beginning teachers of mathematics seek collaboration with other education professionals, parents, caregivers, and community partners to provide the best mathematics learning opportunities for every student.*

Well-prepared beginning teachers of mathematics understand the importance of being a part of a community of educators and believe that the community has the potential to impact teaching in a positive way. "In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for personal and collective professional growth toward effective teaching and learning of mathematics" (NCTM, 2014, p. 99). Beginning teachers anticipate that collaboration with colleagues will spur the need to explain one's teaching practices and articulate rationales for instructional decisions. For example, they are open to making one's ideas and decisions visible and subject to shared scrutiny because they know it will allow them to develop deeper, more widely shared understandings of students' learning (Doerr, Goldsmith, & Lewis, 2010).

Professional learning communities provide teachers with opportunities to collaborate over prolonged periods of time. Five dimensions of successful professional learning communities include (1) common vision and shared values, (2) collective responsibility, (3) leadership support, (4) good facilitation, and (5) use of data and student work (Fulton, Doerr, & Britton, 2010). Observing each other's teaching and providing feedback and protocols for reflecting on practice are often used as key elements in the work of a professional learning community. These dimensions alleviate the isolation of teaching and make it possible for a well-prepared beginner to learn from colleagues and share one's own expertise. For example, during student teaching and other clinical experiences, well-prepared beginning teachers have had opportunities to observe their peers (i.e., other student teachers) and classroom teachers in order to reflect on the effectiveness of specific teaching practices. This reflection is scaffolded by university faculty, supervisors, and school-based mentor teachers who provide prospective teachers with writing prompts and oral questions that enable them to think deeply about what they observed, analyzing both strengths and weaknesses in these situations. Through activities such as these, well-prepared beginning

teachers demonstrate a disposition toward teaching as a collaborative endeavor focused on student learning (Leatham & Peterson, 2010).

Engaging with communities outside of school is another way that teachers can strengthen their teaching. Such communities include families, faith-based organizations, public libraries, local businesses, and community centers that provide additional space and mathematical learning opportunities for students, which teachers can leverage (Aguirre et al., 2012; Civil, 2007; González, Andrade, Civil, & Moll, 2001; Vomvoridi-Ivanovic, 2012). Well-prepared beginning teachers employ multiple strategies to get to know families and communities to better serve students. Examples might include annual home/neighborhood visits with families; organizing a robotics or math club at a local library or community center; meeting with a religious or civic leader serving the basic needs of newcomers in the local community; or volunteering at community based organizations with different language backgrounds to stretch themselves out of their comfort zones.

Well-prepared beginning teachers must be clear and confident in their vision for teaching mathematics. They must be able to effectively communicate this vision while building relationships and trust with families to support mathematics learning throughout the school year. Routine practices such as classroom newsletters, back-to-school nights, family surveys, classroom websites, and parent-teacher conferences can serve as venues for communicating this vision to multiple stakeholders.

Well-prepared beginning teachers work diligently to build relationships with families and caregivers focused on the learning needs of the students. They know it is essential to provide constructive feedback focusing on strengths and areas of growth about students' mathematics performance (Aguirre, Mayfield-Ingram, & Martin, 2013). It is also important to be clear when communicating about learning expectations and homework assignments, including translating letters that are sent home into the parents' language and honoring different strategies that family members might use for doing mathematics. It is important for well-prepared beginners to be ready with strategies that will ensure parents understand the rationale for new innovations in the teaching and learning of mathematics (e.g., new standards or new teaching approaches) and minimize potential fears and concerns that parents might have about these unfamiliar approaches. This could include activities such as organizing family math nights or curriculum nights, sharing specific ways to help with homework, or sending specific family-oriented mathematics activities to parents and caregivers to support their students' learning at home (Hendrickson, Siebert, Smith, Kunzler, & Christensen, 2004; Peressini, 1998). They can also provide resources for family focused mathematics project initiatives such as *Family Math* (Stenmark, Thompson, & Cossey, 1986), *Math and Parent Partners (MAPPS)* (Civil & Bernier, 2006) and *Multicultural Literature as a Context for Mathematical Problem Solving: Children and Parents Learning Together* (Strutchens, 2002), and others, that have shown how engaging with parents as partners in their students' education can increase their children's achievement and advance equity in mathematics education.

Standard C.3: Knowledge of Students as Learners of Mathematics

Well-prepared beginning teachers of mathematics have foundational understandings of students' mathematical knowledge, skills, and dispositions. They also know how these understandings can contribute to effective teaching and are committed to expand and deepen their knowledge of students' as learners of mathematics.

Indicator C.3.1. Students' Thinking about Mathematics Content: *A well-prepared beginning teacher of mathematics attends to students' mathematical thinking and mathematical learning progressions.*

Understanding how students' mathematical ideas develop and connect is at the core of mathematics teaching. Such understanding rests upon knowledge of the mathematics that comes before and after a given mathematics topic (see Standard C.1.4) as well as knowledge about students' informal knowledge, common conceptions, and, where the research knowledge exists, progressions of students' thinking within the content domain. Well-prepared beginning teachers of mathematics have developed strong understandings of students' mathematical thinking in at least one, and preferably more, well-defined content domain(s) (such as within number and operations). They are committed to, and know how to, continue their learning about students' mathematical thinking (e.g., using print or online research/resources, interactions with other professions, and children and their families).

Well-prepared beginning teachers of mathematics can analyze their students' different approaches to mathematical work and respond appropriately. They have the mathematical knowledge and the inclination to analyze written and oral student productions, and look for and identify sensible student mathematical reasoning even when that reasoning may be different from that of the teacher or the student's peers. It is crucial for teachers to have a well-grounded sense of ways that students might think about that content to honor how that work makes sense to them and to think about what instructional moves would best extend students' thinking to question whether it makes sense in all cases or over different contexts (Jacobs, Lamb, & Philipp, 2010). The challenge for well prepared beginning teachers is to place themselves in the shoes of their students and to learn to think like them, rather than immediately correcting their mathematical errors. This knowledge is not only useful in planning for instruction, but also provides a resource for sense-making of moment-to-moment interactions with students.

Well-prepared beginning teachers of mathematics enter classrooms with knowledge of many of the common patterns of student thinking, and know where to access available research-based perspectives on student learning. For beginners, this knowledge is particularly important in content domains that are highly prevalent in the curriculum and those known to be crucial to support students' later mathematical success. For instance, in elementary grades, well-prepared beginners must have facility with a variety of ways in which students make sense of and approach number and operation. They recognize that learning about the ways that students think about and use mathematics is a career-long endeavor. Well-prepared beginners are disposed to – and have skills that enable them to – learn in an ongoing way about students' ways of thinking in many mathematical domains.

Students come to classrooms with unique mathematical perspectives and experiences. Well-prepared beginners know that the quality and focus of their teaching is impacted by the depth and detail of their insight into each student's mathematical thinking. They need to learn about students' informal ideas and invented approaches, as well as students' formal knowledge and understandings. Students' mathematical experiences and resources must also be part of what well-prepared beginners appreciate, make sense of, and build upon to support mathematical learning. Well-prepared beginners are disposed to continually seek information about their students' thinking both because of the breadth of mathematics that they are teaching – knowing how students think about the span of mathematics that

needs to be taught is a substantial undertaking – and because they know that student thinking is continually evolving. They know that learning about a student’s thinking is enhanced by deliberately drawing on the insights of families, professional colleagues, and sources of information from beyond the classroom.

Indicator C.3.2. Students’ Engagement in Mathematical Practices: *Well-prepared beginning teachers of mathematics are able to recognize core mathematical practices within what students say and do across many mathematical content domains, with in-depth knowledge of how students use mathematical practices in particular content domains.*

Because *doing* mathematics is at the core of learning mathematics, well-prepared beginners recognize that students will present a variety of ways for justifying solutions, critiquing the reasoning of others, and approaching problems. As such, they are inclined to look out for and respond to the variety they are presented. Sets of mathematical practices have been usefully elaborated in standards and research syntheses over the past few decades (NCTM, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; NRC, 2001) that can serve as a starting point. Although accomplished teachers have greater breadth of knowing how students engage in mathematical practices across many mathematical content domains, well-prepared beginners know some of the challenges that students may face when engaging in different mathematical practices. For example, that students may accept arguments that have weak mathematical foundations, especially from peers that are their friends or someone they think is typically ‘right.’

Acknowledging that they are at the beginning of their careers, well-prepared beginners have a solid understanding of and how to teach about mathematical practices, and can apply that understanding to some content within their grade-band. They recognize that over years of experience, they will increase their knowledge of students’ ways of using mathematical practices across all content domains. Additionally, they can describe evidence of students demonstrating mathematical practices drawn from multiple content domains. For instance, they can describe how students might reason abstractly and quantitatively about algebra, geometry and statistics.

Indicator C.3.3. Students’ Mathematical Dispositions: *Well-prepared beginning teachers of mathematics know key facets of students’ mathematical dispositions and are sensitized to the ways in which dispositions may impact students’ engagement in mathematics.*

The ability to engage in mathematics must be coupled with an inclination to see such engagement as worthwhile. Teachers need to know if their students, “see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (NRC, 2001, p. 131). Well-prepared beginners know about these facets of disposition and how to gather information about them from their students. This reaches beyond determining if students “like” mathematics and beyond being able to answer students’ “when are we going to use this?” questions. Well-prepared beginning teachers recognize that students

may attribute mathematical proficiency to innate ability or to the application of effort. They know that these dispositions, while strong at times, are not fixed. They also know the ways in which teachers and schools can perpetuate unproductive dispositions. With this knowledge, beginners are prepared to teach in ways that move the needle more toward an attribution of mathematical learning to effort and to extend the meaning and usefulness of mathematics in their students' lives.

Well-prepared beginners have ways of learning about, and fostering, the mathematical dispositions of students that include confidence, flexibility, perseverance, curiosity, self-monitoring, and appreciation of mathematics. They know the impact of their own language, tone, and expectations in influencing dispositions and mathematical self-image of their students. More than just focusing on the kinds of mindsets that students develop in the mathematics classroom, well-prepared beginners realize that perseverance can be supported when students feel a problem is meaningful. Even though they are well prepared, beginning teachers will not initially know the kinds of variations that exist in students' dispositions. They will not initially have many ways to integrate attention to mathematical dispositions into what often appears to be a packed school curriculum. With experience, they will develop the ability to refine their approaches to learning about dispositions of the students they teach, such as tailoring their practices to learn about very young students, students with special needs, and students with cultural backgrounds that differ from their own. They may need support in addressing the tensions that can arise between students' appreciation of mathematics on the one hand and the high stakes nature of mathematical tests that can decrease students' appreciation and confidence.

Standard C.4: Social Contexts of Mathematics Teaching and Learning

Realizing that the social, historical, and institutional contexts of mathematics impact teaching and learning, well-prepared beginning teachers are knowledgeable about and committed to their critical role as advocates for every mathematics student.

Indicator C.4.1. Access and Advancement: *Well-prepared beginning teachers of mathematics recognize the difference between access to and advancement in mathematics learning and work to provide access and advancement for every student.*

Well-prepared beginners know the meaning of access and advancement, understand that denial of access or advancement leads to inequities. Access to mathematics is essential for equitable mathematics education. Access includes ensuring that students have the opportunity to learn important mathematics taught by qualified teachers. Well-prepared beginners recognize that access involves structures in schools and in classrooms and recognize classroom practices that threaten access. Access becomes particularly important in the placement of students into higher-level courses, where the focus is more on doing mathematics, rather than practice. But, access also refers to opportunities within a classroom. Well-prepared beginners realize that access is increased when students can approach a problem from multiple routes, for example, using a method that is familiar to solve the problem, or when they use curriculum materials that include high quality, meaningful tasks and go beyond the narrow curriculum tied to standardized testing. Access is threatened when false hurdles are inherent in the system, for example, denying access to calculators until students master particular skills.

Advancement is the opportunity to go beyond grade-level expectations to learn additional content and the ability to feel whole while doing mathematics. Advancement includes advanced grouping in elementary schools, taking algebra prior to ninth grade, taking advanced or college level courses in high school, and pursuing additional courses or taking honors sections of courses. Well-prepared beginners are prepared to advocate for equitable practices for identifying students for advanced study, recognizing that “success” cannot be defined solely by the teacher or standardized tests, but also the goals that students hold for themselves, especially students who are Black, Latin@, American Indian, emergent multilinguals, and/or students living in poverty (Gutiérrez, Bay-Williams, & Kanold, 2008).

Indicator C.4.2. Mathematical Identities: *Well-prepared beginning teachers of mathematics recognize that their role is to cultivate positive mathematical identities with their students.*

All mathematics teachers are identity workers, in that they contribute to the kinds of identities students are developing both in the classroom and outside of it (Gutiérrez, 2013b). More so than most subjects, students harbor perceptions about what someone who is good at mathematics “looks like;” even very young students can identify who in their classrooms are “good” at mathematics, often times pointing to those that are quick at performing algorithms. Well-prepared beginners know that research and standards provide a different description of what it means to be good at mathematics. For example,

Adding it Up (NRC, 2001) describes a productive disposition as “the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.” Well-prepared beginners seek to actively position all learners as mathematical doers. They understand that developing robust mathematical identities begins with a focus on robust goals for what is important to know and be able to do in mathematics and include doing mathematics for one’s own sake, not just to score well on mathematics tests.

Well-prepared beginners are able to analyze their task selection and implementation, reflecting on the way they may be shaping students mathematical identities. Task selection, such as ones with contexts of baseball, rocket launching, or gendered contexts, may privilege a particular group of students who are familiar and/or engaged in that type of context. Some practices, such as “board races” and timed tests, have long-standing history in U.S. classrooms, despite the fact that they exclude those who need more processing time, while also communicating that those who are fast are good at mathematics. Additionally, well-prepared beginners understand that how their peers and teacher listen to and respect their ideas, has an impact on students developing mathematical identities (Aguirre et al., 2013). For example, asking higher level questions to students perceived by the teacher as more capable, though perhaps unintentional, can negatively impact the development of robust mathematical identities. Similarly, students tuning out someone who they don’t think will share a good strategy, or as students working in groups do not take up the ideas of some of their group members, shapes students’ mathematical identities. Issues of power and privilege arise as students judge the validity of mathematical work based more on racial and gender stereotypes of who is ‘good at math’ rather than on the ideas presented (Esmonde & Langer-Osuna, 2013). Well-prepared beginners view their planning, teaching, and assessment as “identity-in-the-making” (Gutiérrez, 2013a, p. 53), resisting explanations that position the student as inferior or on the margins of the classroom culture when they do not participate in the ways we expect, and instead focusing on how to better support students to develop a robust mathematical identity.

Indicator C.4.3. Students’ Mathematical Strengths: *Well-prepared beginning teachers of mathematics identify and implement practices that draw on students’ mathematical, cultural, and linguistic resources/strengths and challenge policies and practices grounded in deficit-based thinking.*

When teachers are faced with students who think or speak differently from the mainstream, they may inadvertently seek to remedy those differences rather than seeing them as strengths and resources upon which to build. Deficit-based thinking, that assumes students are lacking in something, pervades education policy and practice (Valencia, 2010). Every student enters the classroom with mathematical, cultural, and linguistic strengths that support their learning of mathematics. Well-prepared beginners value and notice these “funds of knowledge” (Moll et al., 1992) and draw upon them in ways that help every learner in the classroom. For example, students from other countries are able to offer algorithms that are mathematically correct but unknown to the US context and can underscore the idea of different approaches to problem solving or different representations of work (Gutiérrez, 2015; Perkins & Flores, 2002).

Well-prepared beginners also realize that supporting the mathematical learning of emergent multilinguals means attending to language and familiar contexts and experiences that promote conjecturing, reasoning, sense-making, and convincing others of mathematical claims so that bilingual

students will be encouraged to use their language skills and become valued members of the mathematics classroom (Dominguez, 2011; Moschkovich, 2012). When working with indigenous students, teachers need to consider how a community's needs along with different ways of knowing may influence representations and the reasons for doing mathematics outside of school (Meaney, Trinnick, & Fairhall, 2013; Wagner & Lunney Borden, 2015).

The beliefs that teachers hold of students can profoundly affect mathematics teachers' rationales for student success and failure as well as the decisions that teachers make to invest in students' learning. Studies that ask teachers to assess the ability of Latin@ and Black students generally show a significant bias towards negative stereotypes and/or low expectations (Baron, Tom, & Cooper, 1985) and many teachers believe the achievement gap is at least partially genetic (Bol & Berry, 2005). Well-prepared beginners are prepared to challenge deficit-based thinking in schools and reflect on their own practice in terms of building upon the cultural, linguistic, and unique ways of knowing of their students.

Indicator C.4.4. Power and Privilege in the History of Mathematics Education: Well-prepared beginning teachers of mathematics understand the roles of power, privilege, and oppression in the history of mathematics education and are equipped to question existing educational systems that produce inequitable learning experiences and outcomes for students.

Schools do not exist in a vacuum; administrators and teachers implement policies and practices based on the historical context in which their schools reside. Therefore well-prepared beginners must be aware of the national, state, district, and school contexts for educating students and be ready to engage in conversations to address inequitable learning experiences. Well-prepared beginners are cognizant of national reform movements in mathematics education, including the strides and challenges in affording every student a quality mathematics education. For example, well-prepared beginners should be familiar with current challenges in mathematics education that acknowledges that too many students who are Black, Latin@, American Indian, emergent multilinguals, and/or living in poverty are not being educated well in mathematics. Well-prepared beginners are abreast of professional standards documents such as the NCTM Standards (2000, 2014), and the National Governors Association Center for Best Practices and Council of Chief State School Officers (2010) and how these documents have evolved to include more (or less) attention to equitable teaching practices (e.g., principles, position statements). At the state and district level, a well-prepared beginner seeks to learn about the demographic trends in their district, segregation and re-segregation policies that result in unequal educational experiences, and the adoption of various textbook and instruction policies. By understanding the historical context of education, well-prepared beginners can learn from past successes and contribute to solving current challenges to advocate for students.

Well-prepared beginners not only know the historical context of mathematics education, they understand that mathematics operates with power and privilege in society. As such, well-prepared beginners are knowledgeable of mathematics education as part of a broader system of mechanisms that determine what is valued, what is right and what is normal in society (Valero, 2009). While expert teachers of mathematics are well grounded in literature that offers strategies for transforming schooling for students who have historically been denied access to a quality mathematics education and implement these practices in their classrooms and schools (e.g., Berry, 2008; Gutstein, 2006; Leonard & Martin, 2013; Rodriguez & Kitchen, 2005), well-prepared beginners have read and know how to access

such literature and recognize the importance of implementing practices that empower every student. They are prepared to ask questions to understand current policies and practices, and to raise awareness of potentially inequitable practices. This is particularly important related to students who are Black, Latin@, American Indian, emergent multilinguals, and/or students living in poverty because they are overrepresented in classrooms that emphasize skill-based instruction, worksheets, and computer programs instead of problem-solving lessons and tasks (Oakes, 2008). For example, well-prepared beginners might ask how students are recommended and placed in gifted and remedial/intervention programs; whether the placement of students across various programs is representative of the school population; who determines the type of instructional materials that are available to students to support their learning of rigorous mathematics; and how information regarding various mathematics programs is communicated to parents.

Indicator C.4.5. Ethical Practice for Advocacy: Well-prepared beginning teachers of mathematics are knowledgeable about and accountable for enacting ethical practices that allow them to advocate for themselves and to challenge the status quo on behalf of their students.

The Council for the Accreditation of Educator Preparation (CAEP), Mathematics Teacher Education Partnership (MTE-Partnership), and others cite the importance of teacher dispositions, including ethical practice. To develop into accomplished teachers who impact not just education but also the lives of their students, well-prepared beginners must engage in ethical practices and advocacy. Ethical practice means that professional decisions and actions are guided by a set of principles around mutual respect, integrity, and a sense of justice, not simply external measures of quality teaching like students' test scores or teacher evaluations. Well-prepared beginners exercise ethical practice in decision-making and teaching when working with children, families, and colleagues, including the elimination of deficit-based thinking, integration of family/community funds of knowledge into lesson design and implementation, attention to cultural differences, and protection of the rights of privacy.

Well-prepared beginners recognize the need to challenge previous functions of school mathematics and the messages school mathematics convey to students and the greater public. Included in the messages to challenge are: "being good at mathematics is a sign of intelligence;" "some students are good at mathematics while others simply are not;" "mathematics is a natural/pure way of doing things;" "doing mathematics does not involve emotions, values, or the body;" and "all students should aspire to STEM careers." Well-prepared beginners recognize an advocacy role in teaching, realizing that when teachers fail to take action, students, families, colleagues, and others may be harmed.

To address this indicator, well-prepared beginners demonstrate the understanding that teachers who are successful advocates for students challenge the status quo by developing a principled and just ethical practice that holds themselves and others accountable. Rather than simply reflecting on or analyzing teaching, well-prepared beginners accept that teaching is a political activity (Schoenfeld, 2010) and are prepared to take action in the classroom, at school meetings, in professional settings, and other spaces in order to advocate for meaningful and robust mathematical student identities and experiences. This knowledge prepares well-prepared beginners to develop language and effective ways of working with allies, choosing one's battles appropriately, and being creative and strategic in response to practices and policies that demean students and teachers. Teachers who successfully advocate for students realize that teaching sometimes requires acts of creative insubordination (Gutiérrez, 2015).

That is, driven by higher ethics, successful beginning teachers are prepared to re-interpret school rules and practices when not in the best interest of their students having meaningful and humane mathematical experiences.

Summary

This chapter described four equally important standards (with sets of indicators) that describe the knowledge, skills, and dispositions that well-prepared beginners have upon completion of their preparation programs. These standards are not discrete lists, but are inter-related and inter-dependent. For example, one cannot support the mathematical learning for each student, without having the knowledge of content, pedagogical skill, and awareness of social contexts. Setting such high expectations for beginning teacher of mathematics is critical to the success of K-12 students. To reach these goals for candidates requires strong preparation programs, the focus of Chapter 3.

While examples from a particular grade-band may be included to help explain a standard or indicator, Chapters 4 through 7 will provide additional explanation, as appropriate, to address the specialized needs for preparing mathematics teachers at a given grade level.

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CHAPTER 3: PROGRAM CHARACTERISTICS AND QUALITIES

Teacher preparation programs can take various forms, such as an undergraduate degree program in education (or a related field) leading to initial certification or licensure, a fifth-year program (possibly leading to a graduate degree) designed for persons with relevant undergraduate degrees to gain initial certification or licensure, and non-degree programs offered by institutions of higher education, school districts, or other providers designed to attain initial certification or licensure. Moreover, these programs may be specific to mathematics (often focusing on the middle or secondary grades) or may lead to more general certifications that include mathematics (such as early childhood or elementary grades, as well as special education.) Whatever the structure of the program, it should meet the program standards described in this document in order to ensure that its completers attain the standards of knowledge, skills, and dispositions needed by a well-prepared beginning teacher of mathematics.

Chapter 2 described the knowledge, skills and dispositions needed by well-prepared beginning teachers of mathematics. This chapter describes standards for preservice programs to ensure that their students meet those standards.

Organization of this Chapter

This chapter describes the characteristics and qualities of effective mathematics teacher preparation programs in five sections. The first discusses the important role of partnerships among stakeholders in ensuring the effective preparation of mathematics teachers. The second describes the opportunities to learn mathematics for teaching that need to be provided to candidates, while the third describes the opportunities to learn to effectively teach mathematics that need to be provided to candidates. The fourth addresses candidates' opportunities to learn in clinical settings. The fifth discusses effective practices for recruiting (and retaining) candidates into mathematics teacher preparation.

Each standard includes a number of more-specific indicators for that standard, along with accompanying explanations. These standards and indicators apply for all well-prepared beginning teachers of mathematics from prekindergarten through high school. While examples from a particular grade-band may be included to help explain a standard or indicator, Chapters 4 through 7 will provide additional explanation, as appropriate, to address the specialized needs for preparing mathematics teachers at a given grade level.

Table 3.1. Standards for Effective Programs for Preparing Beginning Teachers of Mathematics and Related Indicators

Standard	Related Indicators
Standard P.1: Establish Partnerships <i>Effective programs for preparing mathematics teachers have significant input and participation from all appropriate stakeholders.</i>	P.1.1. Engage All Partners Productively P.1.2. Provide Institutional Support
Standard P.2: Opportunities to Learn Mathematics <i>An effective mathematics teacher preparation program provides beginning teachers with opportunities to learn mathematics that are purposefully focused on essential big ideas across content and processes that foster a coherent understanding of mathematics for teaching.</i>	P.2.1. Students' Thinking about Mathematics Content P.2.2. Build Mathematical Practices and Processes P.2.3. Provide Sustained Experiences
Standard P.3: Provide Opportunities to Learn to Teach Mathematics <i>An effective mathematics teacher preparation program provides candidates with multiple opportunities to learn how to teach through mathematics-specific methods courses that integrate mathematics, knowledge of students as learners, the social contexts of mathematics teaching and learning, and practices for teaching mathematics.</i>	P.3.1. Address Deep and Meaningful Mathematics Content Knowledge P.3.2. Provide Candidates with Strong Foundations of Knowledge about Students as Mathematics Learners P.3.3. Address the Social Contexts of Teaching and Learning P.3.4. Provide Practice-Based Experiences P.3.5. Provide Effective Mathematics Methods Instructors
Standard P.4: Opportunities to Learn in Clinical Settings <i>The clinical experiences of effective teacher preparation programs are guided by a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for teacher candidates to teach and to learn from their own teaching and the teaching of others.</i>	P.4.1. Collaboratively Develop and Enact Clinical Experiences P.4.2. Sequence School-Based Experiences P.4.3. Experience Teaching with Diverse Learners P.4.4. Recruit and Support Qualified Mentor Teachers and Teacher Preparation Supervisors P.4.5. Ethical Practice for Advocacy
Standard P.5: Recruit and Retain Teacher Candidates <i>An effective mathematics teacher preparation program attracts, nurtures, and graduates high quality teachers of mathematics who are representative of diverse communities.</i>	P.5.1. Recruit Strong Candidates P.5.2. Address Diverse Community Needs P.5.3. Provide Experiences and Support Structures P.5.4. Assess Recruitment and Retention Data

Standard P.1: Establish Partnerships

Effective programs for preparing mathematics teachers have significant input and participation from all appropriate stakeholders

The Council for the Accreditation of Educator Preparation (CAEP) (2015) has described the need for and characteristics of effective partnerships. The Association of Public Land-Grant Universities' (APLU) Mathematics Teacher Education Partnership has developed *Guiding Principles for Secondary Mathematics Teacher Preparation Programs* (Mathematics Teacher Education Partnership, 2014) to provide guidance to partner institutions that seek to educate secondary mathematics teachers. These and other documents can help guide the establishment of the partnership.

Strong partnerships engage all partners in developing a common vision and identifiable goals. As argued by the National Association for Professional Development Schools (2008), a common vision should focus on “the advancement of the education profession and the improvement of P - 12 learning” (p. 3). More specifically, this vision should lead to identifiable goals that promote the “professional growth across the continuum of preservice teacher candidates, in-service educators, and college/university faculty and administrators” (p. 4).

While members of the partnership may include a broad range of stakeholders, it is imperative the partnership include mathematics teacher educators and researchers, their teacher preparation colleagues, P-12 educators and administrators, mathematicians, and community-based organizations and community members. Leadership for creating and sustaining the partnership will typically fall to the mathematics teacher educator.

We envision this group working within a professional council of stakeholders. This council is responsible for reviewing the partnership and verifying that the partnership is inclusive of all stakeholders. Partners recognize that for a collaboration to be successful, it must have systematic ways of verifying that the partnership is mutually beneficial.

Indicator P.1.1. Engage All Partners Productively: *An effective teacher preparation program benefits from an interdisciplinary collaborative partnership that is a shared endeavor focusing on the preparation of mathematics teachers who are well prepared to improve PK-12 student learning in mathematics from multiple perspectives.*

An effective teacher preparation program benefits from an interdisciplinary collaborative partnership that is a shared endeavor focusing on the preparation of mathematics teachers who are well prepared to improve PK-12 student learning in mathematics from multiple perspectives. These types of partnerships inform different facets of mathematics teacher preparation based upon the wisdom of practice and solid theoretical and research-based knowledge and do so in concert with practitioners so as to prepare effective mathematics teachers for work in diverse schools.

Mathematics teacher educators (MTEs) provide leadership to ensure that the partnership’s vision recognizes the complex nature of mathematics teaching and learning and respects the contributions needed from each partner. MTEs play a central role in assuring the program supports the preparation of

well-prepared beginners. The partnership is enhanced by the contributions of research scholars who insure that the program is designed using the best available knowledge.

The partnership includes active participation by **faculty who teach mathematics and statistics** courses. *The Mathematical Education of Teachers II* (MET2) has argued that “teacher education should be recognized as an important part of a mathematics department’s mission and should be undertaken in collaboration with mathematics education faculty,” that “Prospective teachers need mathematics courses that develop a solid understanding of mathematics they will teach” and that mathematics department should offer a minimum of “three courses with a primary focus on high school mathematics from an advanced viewpoint.” When this happens, an institution’s mathematics teacher educators are positioned to partner with mathematics (and statistics) faculty to ensure both mathematics courses and mathematics education courses that are designed to educate well-prepared beginning teachers.

An effective partnership requires **collaboration with faculty** in social foundations, special education, educational psychology, educational leadership, and learning technologies as well as faculty in other disciplines who teach courses in teacher preparation.

The engagement of **P-12 school-based personnel** is essential to the partnership. Through shared responsibility, mathematics teacher preparation programs can integrate coursework, theory and pedagogy. The partnership should ensure that future teachers have high quality school-based experiences that are needed to educate well-prepared beginning teachers and support the development of candidates’ skills as related to the needs of schools and school districts. Similarly, close cooperation with preparation programs helps districts hire teachers who are better prepared to be effective in their schools. Building these partnerships not only supports candidate’s learning, it can deepen classroom teachers’ knowledge of mathematics content and pedagogy to support their students’ learning.

Families and community leaders are an important but often overlooked participant in teacher preparation partnerships. When mathematics teacher educators collaborate with families and leaders in the community, they can design learning experiences that help prepare teachers to better understand: family and community cultural perspectives, the various activities and responsibilities students have in their homes and communities, the kinds of mathematics that is performed by community members in their jobs, and the kinds of values that are highly regarded. In this way, mathematics teacher educators can help the beginning teacher to reflect on and build lessons and classroom cultures that support students to “be themselves” and “be experts” on things that others in the classroom (including the teacher) may not.

Partnerships should seek to partner with families and community leaders to create activities that are immediately and mutually beneficial for the beginning teacher and students. Such activities could include students learning mathematics with and from beginning teachers and others in community spaces (e.g., public libraries, Boys & Girls clubs, community centers, or places of worship).

Indicator P.1.2. Provide Institutional Support: *Institutional commitment to a strong mathematics teacher education program includes institutional support for the mathematics teacher educator’s career trajectory and appropriate rewards for both their institution and school-based partners.*

Educator preparation programs that aspire to have a high-quality teacher education program must provide the resources and the support needed to achieve this vision. In particular, it is important that the preparation program's reward structures, including awards, commendations, salaries, and promotion and tenure clearly support this work. Mathematics teacher preparation cannot be viewed as a duty or chore delegated to graduate assistants or instructors, but rather a core activity of all involved departments.

Institutions must support program improvement activities, including supporting mathematics teacher educators' attendance at relevant professional conferences so they may learn about other high quality programs. For example, attending the annual meeting of the Association of Mathematics Teacher Educators (AMTE) and/or the Council of Exceptional Children (CEC) Teacher Education Division (TED) and/or the Professional Development Schools (PDS) conference could expand faculty members' awareness of effective program components.

Institutions must provide resources necessary to support teaching and learning. Mathematics teacher educators need access to the materials vital to teaching mathematics in P-12 schools, to prepare beginning teachers for the classrooms in which they will teach. For example, faculty teaching a mathematics methods course should have access to textbooks, online resources, and other instructional materials used by P-12 teachers in their area, to help beginning teachers learn how to effectively use curriculum materials.

Education Preparation Providers (EPPs) must value the work involved in developing and improving mathematics teacher education programs. Such work includes communication and collaboration with stakeholders, program assessment work, and work related to program reviews and meeting program standards. This important work should be valued in the promotion and tenure process.

Standard P.2: Provide Opportunities to Learn Mathematics

An effective mathematics teacher preparation program provides beginning teachers with opportunities to learn mathematics that are purposefully focused on essential big ideas across content and processes that foster a coherent understanding of mathematics for teaching.

Developing a teacher's Knowledge of Mathematics for Teaching² must be a priority in a teacher preparation program, and developing such knowledge is a career-long endeavor. Knowledge of Mathematics for Teaching extends beyond general subject matter knowledge. It includes the knowledge of mathematics that teachers need to design and adapt lesson plans, anticipate student questions and misconceptions, evaluate student work, use proper notation and vocabulary, and the knowledge of how mathematics content is connected across grade levels in order to convey mathematical ideas accurately, anticipating how these ideas develop over time.

² As addressed in the preface, this document will use the term *mathematics* to encompass mathematics and statistics, because school mathematics teachers are responsible for instruction in both content areas of mathematics and statistics.

Within a mathematics teacher preparation program, a variety of individuals will provide opportunities for prospective teachers to learn mathematics content. Regardless of their departmental affiliation or academic background, whether they are employed by a university or a school district, they should be considered part of the community of mathematics teacher educators. The responsibility for ensuring appropriate opportunities for prospective teachers to learn mathematics is the joint responsibility of all such mathematics teacher educators.

The NCTM/CAEP Standards (NCTM, 2012a; NCTM, 2012b) describe specific content requirements for preservice teachers of secondary mathematics, and *Mathematical Education of Teachers II* (MET II; CBMS, 2012) and *Statistical Education of Teachers* (SET) (Franklin et al., 2015) provide specific guidance on the mathematics courses required for teaching at the elementary, middle, and high school levels, which are summarized in the following table. **We take these recommendations as the minima for effective mathematics teacher preparation.** More detailed discussion of the mathematics content expectations is provided in chapters 4 through 7 of this document.

Table 3.2. Minimum Mathematics Content Preparation for Teacher Candidates
(CBMS, 2012; Franklin et al., 2015)

PK-Grade 5	<p>MET II: 12 semester-hours focused on a careful study of mathematics associated with the CCSS (K–5 and related aspects of 6–8 domains) from a teacher’s perspective.</p> <p>SET: A minimum of six weeks of instruction included in the coursework.</p>
6-8	<p>MET II: Mathematics and statistics courses designed for future middle grades teachers</p> <ul style="list-style-type: none"> Number and operations (6 semester-hours) Geometry and measurement (3 semester-hours) Algebra and number theory (3 semester-hours) Statistics and probability (3 semester-hours) Other mathematics and statistics courses (9 hours) Introductory statistics (recommended) Calculus Number theory Discrete mathematics History of mathematics Modeling <p>SET: 2 courses in statistics (could be combined with the above)</p>
High School	<p>MET II: The equivalent of an undergraduate major in mathematics that includes three courses with a primary focus on high school mathematics from an advanced viewpoint.</p> <p>SET: 3 courses in statistics that include a data-analytic and simulation-based approach with a focus on statistical models and inference.</p>

Indicator P.2.1. Attend to Mathematics Content for Teaching: *An effective mathematics teacher preparation program provides opportunities for prospective teachers to learn, with understanding and depth, the school mathematics content they will teach.*

Opportunities to learn mathematics content should be **required of all who are preparing to teach** mathematics or to support student learning of mathematics. That is, in addition to early childhood, elementary, middle grades, and secondary mathematics teachers, these standards are directed towards general education teachers at the early childhood and elementary levels, special education teachers, teachers of English Language Learners, mathematics specialists, and all others who will have responsibility for aspects of student learning in mathematics. However, the preparation needs for these particular specializations will vary. We echo the recommendation of the MET II report (CBMS, 2012, p. 37): “Special education teachers and ELL teachers who have direct responsibility for teaching mathematics (a core academic subject) should have the same level of mathematical knowledge as general education teachers in the subject.” As with all teachers, they must understand not only the mathematics they will teach at a deep level, but also mathematics before and after that grade (CBMS, 2012).

Programs that focus on the mathematical content knowledge of beginning teachers of mathematics are directly addressing the issue of equity. All students deserve teachers who possess the content knowledge they need to teach well. By ensuring that those who complete teacher preparation programs have strong content knowledge, understanding of the practice of mathematics, and positive mathematics identities, programs are promoting a teaching workforce that provides an equitable education for all students. Of course, this is only achievable if it is accompanied by the perspective that the purpose of teaching mathematics is to meet the needs of all learners. Too often, mathematics is used as a “sorting” subject, one that separates those who can from those who can’t. That belief has neither a place in content courses nor other content-focused experiences for teachers. Courses should be taught in a way that engenders positive mathematics identities.

Indicator P.2.2. Build Mathematical Practices and Processes: *An effective mathematics teacher preparation program provides opportunities to learn mathematics that enable prospective teachers to engage in mathematical practices and processes that are appropriate to the content being studied.*

Mathematics includes more than learning appropriate content. Programs ensure that candidates are immersed in mathematical practices and processes of reasoning, sense-making, and problem solving. Prospective teachers’ mathematical experiences include continued emphasis on reasoning abstractly and quantitatively, explaining their thinking, and analyzing the thinking of others. When the content lends itself to such practices and processes, opportunities to learn include seeing structure and generalizing. Mathematical modeling—using mathematics to analyze real-world situations—receives continued attention throughout the program. Programs also provide explicit opportunities to understand statistical ways of thinking and to understand the different habits of mind in both mathematical thinking and statistical thinking (Chance, 2002).

Effective programs provide specific opportunities for prospective teachers to engage in mathematics using technology. For example, visualizations of a data set can help students better understand patterns within a data set, and spreadsheets can help students create models of mathematical situations, thus supporting the development of mathematical practices and processes. “Mathematical action technologies” that “perform mathematical tasks and/or respond to the user’s actions in mathematically defined ways” (Dick & Hollebrands, 2011, p. xii) support the development of candidates’ deep conceptual understanding through use of mathematical practices and processes.

Professional organizations representing the mathematics community recognize the benefits of active learning in mathematics (e.g., MET II; MAA’s forthcoming Instructional Practices Guide). Active learning in STEM disciplines improves learning (Freeman et al., 2014). In such settings, learners are typically provided challenging tasks that promote mathematical problem solving and provide opportunities to discuss their thinking in small and full-group discourse, thus promoting important mathematical practices (Webb, 2016). Effective programs structure opportunities to learn so that prospective teachers learn mathematics by using these active learning strategies. In this way, prospective teachers experience learning mathematics using methods that are consistent with the methods they should use as teachers.

Mathematics should be taught in a way that makes the instructor’s choices about how to address mathematical practices and processes transparent. Often undergraduate students (in this case prospective teachers) only focus on the mathematical products without consideration of the underlying mathematical practices leading to those products. Mathematics instruction should emphasize the development of those mathematical practices. Mathematics teacher educators should explicitly identify and address these mathematical practices for those learning to teach mathematics.

Effective programs focus on building mathematical practices and processes in a manner that honors the social context of teaching and learning mathematics. Those who teach mathematics courses in effective programs must model equitable practices, including making apparent underlying beliefs of the role of each individual in the classroom community. For example, mathematics as a discipline values “argument” as the way to find mathematical truth, but society does not always value argument from boys and girls equally. Without explicit classroom attention to mathematical argumentation, effective programs risk diminishing the opportunities prospective teachers have to engage in the mathematical practice of “constructing viable arguments.” Prospective teachers who have been successful in the prevailing mathematical culture should understand that though mathematics seems to be objective, bias is inherent because it is a human endeavor.

Indicator P.2.3. Provide Sustained Experiences: *An effective mathematics teacher preparation program provides multiple opportunities to learn mathematics for teaching in varied settings across the program.*

Mathematics content experiences should be in-depth, focused, and enacted in different settings; they should not be diluted, superficial, or limited solely to college classrooms. For example, assigning one course or even a sequence of courses as an isolated experience does not fulfill the spirit of this standard; engagement with rich content opportunities should be coupled with ongoing integrated learning experiences in mathematics. Programs should acknowledge that a deep understanding of mathematics content and practice includes a sustained focus on mathematical ideas throughout all aspects of the program. Mathematics methods or pedagogy courses should be specifically focused on the teaching of

mathematics with clear attention to accurate mathematics content and mathematical practices and processes (i.e., ways of doing mathematics). Field experiences and induction experiences should also include a focus on the teaching of important mathematical content where content-specific rubrics are used to evaluate lessons in observations by mentor teachers and university supervisors. In addition, mathematics content courses should be taught using mathematics teaching methods (such as those discussed in Standard 2) that serve as models of effective mathematics teaching for prospective teachers.

Effective programs instill in beginning teachers the understanding that they will continue to learn mathematics throughout their careers as they make the transition from learning mathematics to joining the profession as teachers of mathematics. Effective programs provide prospective teachers opportunities to learn how to learn mathematics by studying mathematics curriculum materials (Davis & Krajcik, 2005). In addition, effective programs provide opportunities for prospective teachers learn how to learn mathematics in collaboration with their colleagues as a part of a learning community (NCTM, 2014).

Opportunities to learn mathematics for teaching may come in mathematics courses and may also be included in other aspects of the program, including clinical experiences, methods courses, involvement in professional mathematics education organizations (local, state, or national), tutoring, service learning, clubs, or research experiences, when these aspects are focused on school mathematics content.

Standard P.3: Opportunities to Learn to Teach Mathematics

An effective mathematics teacher preparation program provides candidates with multiple opportunities to learn how to teach through mathematics-specific methods courses that integrate mathematics, knowledge of students as learners, the social contexts of mathematics teaching and learning, and practices for teaching mathematics.

Although many configurations for programs address opportunities to learn to effectively teach mathematics, we use the phrase *mathematics methods courses* to refer to specific types of courses that are neither mathematics courses nor general pedagogy/methods courses, but instead lie at the intersection and focus on the pedagogy associated with teaching mathematics. As such, mathematics methods courses must take into account the particular nature of the discipline of mathematics (Conference Board of Mathematical Sciences, 2012) as well as issues associated with effective teaching of mathematics. Opportunities to deeply learn fundamental mathematics simultaneously with issues of pedagogy are critical for prospective mathematics teachers (Steele & Hillen, 2012). Second, the first four indicators in this section are components of mathematics methods courses, and although they are listed separately, they should be addressed in an integrated manner.

Indicator P.3.1. Address Deep and Meaningful Mathematics Content Knowledge: *An effective mathematics teacher preparation program provides mathematics methods courses associated with a rich view of mathematics.*

Because many candidates enter methods courses without having deeply conceptualized all of the mathematics they will be responsible to teach, mathematics methods courses must present multiple opportunities for candidates to reconsider and deepen their mathematical understandings as both learners of mathematics and as mathematics teachers. Although mathematics-based methods courses are preferred to generic methods courses, if the mathematics addressed is solely or primarily procedurally oriented, then all of the other worthy goals of the methods course will be subservient to propagating a widely-held practice that mathematics, instead of fundamentally being about engaging in ways of reasoning about quantitative and spatial concepts and principles, is instead fundamentally about learning to memorize mathematical facts and carry out procedures (Hiebert et al., 2005).

Teacher candidates must not only learn mathematics concepts and procedures, but they must also develop productive mathematical dispositions. Teacher candidates come with years of experience as mathematical learners and some may hold unproductive beliefs about mathematics and mathematics teaching and learning. Therefore productive mathematical disposition is the oft-missing strand of mathematical proficiency, and mathematics methods courses must support candidates in developing richer and more positive mathematical dispositions.

Mathematical tasks are central to mathematical reasoning, and rich mathematical tasks emphasizing high cognitive demand (Stein, Smith, Henningsen, & Silver, 2000) must be an integral component of mathematics methods courses. However, even rich mathematical tasks are often rendered devoid of most of their mathematical richness when teachers over-scaffold, thereby leading students through the task without focusing on the underlying thinking and reasoning that the tasks were intended to evoke. Candidates in mathematics methods courses must not only engage in rich mathematical tasks that are implemented in ways designed to sustain the cognitive demand, but they must also learn to successfully implement high-level tasks in PK-12 settings (Smith, Bill, & Hughes, 2008).

Indicator P.3.2. Provide Candidates with Strong Foundations of Knowledge about Students as Mathematics Learners: *An effective mathematics teacher preparation program provides extensive experiences focusing on students as mathematics thinkers and learners.*

A main goal of mathematics methods course is to develop candidates' understanding of how students think and learn about mathematics (see Standard 2.3). Methods courses should provide multiple experiences (e.g., readings, analyses of videos, conducting teaching experiments) that develop deep understanding of at least one (or two) research-based learning trajectories' developmental progressions (e.g., sequences of these patterns of thinking in a domain). For example, the methods course would provide extensive, complementary experiences focusing on how students think and learn about a key topic such as whole number and operations for early childhood; fractions for elementary mathematics education; proportional reasoning or algebra for middle school mathematics teachers; and geometry, for high school mathematics teachers. These experiences also develop competencies in connecting these progressions to specific implications for instruction—that is, learn a complete learning trajectory—and connecting content to mathematical practices.

Mathematics methods courses should simultaneously provide candidates with experiences to develop strategies for understanding and building students' (a) productive dispositions and positive math identities and (b) meaningful mathematical sense making and use of mathematical practices.

Indicator P.3.3. Address the Social Contexts of Teaching and Learning: *Mathematics methods courses provide candidates with foundational knowledge about the social, historical, political and institutional contexts that affect mathematics teaching and learning. By closely examining these contexts and the structures, policies, and practices that foster and constrain student access to and advancement in mathematics, candidates develop a deeper understanding and ethical skill set for advocacy work in mathematics education.*

Mathematics methods courses provide candidates with foundational knowledge about the social, historical, political and institutional contexts that affect mathematics teaching and learning. By closely examining these contexts and the structures, policies, and practices that foster and constrain student access to and advancement in mathematics, candidates develop a deeper understanding and ethical skill set for advocacy work in mathematics education.

Many teacher preparation programs include specific courses designed to address social contexts of teaching and learning (e.g. multicultural education), wherein candidates grapple with a variety of equity issues, including examining the roles that power, privilege and oppression play in schooling (e.g., tracking) as well as effective anti-racist and social justice pedagogies that disrupt institutional bias with teaching innovation, critical reflection, and social action. However, it is important to also explicitly address equity issues within mathematics methods courses. For example, mathematics methods courses should include critical analyses of current mathematics education systems, including the histories and the institutional tools, policies and practices that shape what mathematics gets taught, how, and to whom. This is essential because the current mathematics education system is unjust and grounded in a legacy of segregation, systems of power and privilege, and deficit-thinking based on race, ethnicity, class, language and gender (Berry et al., 2014; Martin et al., 2010).

There is additionally a need for well-prepared beginning teachers to challenge deficit views about learning by questioning the status quo at a systemic level. For example they should consider testing and tracking systems that have instituted ways to identify, label, and separate children by perceived mathematics abilities (Cogan et al., 2001; Oakes, 2005). Methods courses should provide candidates with tools and frameworks to support a more asset- and resource-based instructional approach that focuses on students' strengths in learning.

Mathematics methods courses must prepare teachers to navigate the mathematics education political terrain. The unique high stakes-high status context of mathematics education puts extreme pressure on teachers that is consequential to mathematics learning, performance, and student affect. Along with cross-disciplinary coursework, mathematics methods courses must provide a foundation for new teachers to recognize, navigate, and begin to understand the challenges associated with ultimately transforming these political contexts into a more just and equitable mathematics education for our nation's youth (Gutierrez, 2013a).

Mathematics methods courses must prepare beginning teachers to recognize the key role identity and power plays in mathematics education. As identity workers, teachers have tremendous power in how children, their families, and communities see students as doers of mathematics (Gutierrez, 2013b). Mathematics methods courses offer ways for teacher candidates to critically assess their students' mathematics identities and create learning opportunities to strengthen those mathematics identities in positive ways.

Mathematics methods courses must provide opportunities for teacher candidates to learn about and build on the multiple mathematical, cultural, linguistic, and family strengths that students bring to the classroom. This means going beyond traditional field placements and into community settings to learn from and about students, families and communities. Viewing these social, cultural, and community contexts as resources, rather than barriers, for mathematics teaching and learning needs explicit emphasis in mathematics methods courses (Aguirre et al., 2013; Civil, 2007; Foote, 2009).

Developing an ethical practice for advocacy in mathematics education starts with a strong foundation set in mathematics methods courses and their field experiences. The concept of ethical practice and the opportunity for beginning teachers to develop their own stance, through such tools as The Mirror Test - a series of critical reflection questions about ethical obligations for doing right by students (Gutiérrez, 2016). Moreover, teacher preparation programs should assess the ethical practice beginning teachers need to inform and improve mathematics instruction.

The development of this ethical practical for advocacy cannot be accomplished in isolation. It necessitates collaboration with multiple communities, including face-to-face and virtual communities, that can provide needed resources, advice, models and emotional support to engage in this demanding work. Methods courses must provide candidates multiple opportunities to develop knowledge and skills necessary for an ethical practice to take action and advocate for students in multiple ways and various settings.

Indicator P.3.4. Provide Practice-Based Experiences: *Mathematics methods courses provide candidates with practice-based experiences using tools and frameworks grounded in research to develop core pedagogical practices and pedagogical content knowledge for teaching mathematics.*

Effective mathematics methods courses employ a practice-based approach (Ball & Cohen, 1999). The decomposition of practice movement (Grossman et al., 2009) engages candidates in detailed activities set in the everyday work of teaching. In this approach, authentic artifacts of practices such as mathematical tasks, lesson plans, curricula, student work, and classroom episodes in the form of narrative or video cases are used and analyzed through various lenses. This practice-based notion of teacher education is different from the traditional view in that teachers experience the emergence of theories from analyses of practice rather than separating the learning theories from the later application of these theories to practice (Smith, 2001).

In the mathematics methods course, research-based frameworks, tools, and strategies serve as important vehicles for connecting theory and practice and guiding candidates in their work with authentic artifacts of teaching. For example, when focusing attention on key elements of planning a lesson, the methods instructor strategically uses planning tools that emphasize building on students' thinking and orchestrating a productive discussion (Smith, Bill, & Hughes, 2008), teacher questioning, differentiating instruction, and culturally responsive instruction (Bay-Williams, McGatha, Kobett, & Wray, 2014).

Collaborating with clinical experience coordinators is an effective way for methods instructors to connect their course experiences with PK-12 classrooms and children. Professional development schools are an excellent environment for collaboration. Having experiences in clinical settings prior to the student-teaching experience helps teacher candidates situate their learning in practice.

These classroom-based experiences can be enhanced through the careful selection of artifacts of practice, including the use of video and classroom-based cases. These selections need to offer illustrations of practice and learning that capture real teachers teaching real children in real classrooms, including students from various backgrounds, abilities and understandings doing powerful mathematics and utilizing productive strategies, thereby challenging negative racial and gender stereotypes as well as fixed mindsets about who is capable of engaging in rich and rigorous mathematics. When the selection of examples is broadened to classroom-based examples that show students learning mathematics in multiple languages (Celedon-Pattichis & Ramirez, 2012; Moshkovich, 1999, 2002) and cultural and community experiences (Aguirre & Bunch, 2012), then programs make clear statements about not only how mathematics teaching can be effective, but also for whom.

Indicator P.3.5. Provide Effective Mathematics Methods Instructors: *Effective mathematics teacher preparation programs ensure that mathematics methods instructors have relevant grade-level experience with schools, teachers, and students and possess a deep understanding of the mathematics content and of the research and practice regarding pedagogical and equity issues associated with effectively teaching all students.*

A mathematics methods course that addresses the Indicators 3.3.1 through 3.3.4 provides complex, rich experiences drawing upon a deep view of mathematics and a broad understanding of the mathematics education literature base. Thus, mathematics methods instructors must possess mathematical knowledge, pedagogical content knowledge and the knowledge of social cultural contexts of mathematics. Further, methods instructors must also take a stance of learning from practice and research that will enable them to keep learning and prepare them to support candidates to continue learning from teaching beyond the methods courses as an integral part of their professional journey (Hiebert et al., 2007). For example, new research about students' thinking in well-defined content domains, advances in educational technology available for mathematics classes, and changing standards require mathematics methods instructors to take a professional stance, that is, view teaching as constant learning. Furthermore, given the complex demands of teaching mathematics methods, programs must consider how to support mathematics methods instructors as they develop the relevant grade-level school experience and update their knowledge of research and practice. Also, institutions of higher education that prepare university faculty must explicitly and programmatically support the preparation of methods course instructors to meet these demands.

Standard P.4: Opportunities to Learn in Clinical Settings

The clinical experiences of effective teacher preparation programs are guided by a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for teacher candidates to teach and to learn from their own teaching and the teaching of others.

Clinical experiences are guided, hands-on, practical applications and demonstrations of professional knowledge of theory to practice, skills, and dispositions through collaborative and facilitated learning in field-based assignments, tasks, activities, and assessments across a variety of settings. Increased calls to

embed clinical experiences in real contexts of teaching point to the importance of learning through engagement in teaching (Ball & Forzani, 2009; Grossman et. al., 2009; Lampert & Graziani, 2009). Furthermore, clinical experience along with content knowledge and the quality of the prospective teachers are the components of teacher preparation that are likely to have the strongest effects on outcomes for students (National Research Council [NRC], 2010).

An effective mathematics teacher preparation program provides clinical experiences that are developed mutually with school partners, are scaffolded to build in complexity, include opportunities to work in diverse settings and with a range of learners, and are supervised by qualified mentors. Each of these four critical aspects of clinical experiences is described in this section.

Indicator P.4.1. Collaboratively Develop and Enact Clinical Experiences: *An effective mathematics teacher preparation program collaborates with school partners to enact a shared vision of effective mathematics teaching.*

For candidates to be well prepared in the teaching of mathematics, their programs must provide consistency in what is being taught and modeled in courses and field experiences. Effective mathematics teacher preparation programs work to establish a shared vision and support systems among faculty, supervisors, mentor teachers, and teacher candidates focused on enacting effective mathematics teaching practices (as described in Chapter 2). Through their collaboration, school and university partners develop shared language to discuss teaching and learning, as well as co-create needed routines, tools, and norms necessary for achieving that vision.

Beyond establishing shared expectations, effective programs have reciprocal professional relationship among stakeholders (university faculty and supervisors, mentor teachers, and school-based personnel) that are integral to the design, implementation, and ongoing assessment of the preparation program. This might include co-designing field-based assignments and/ or co-teaching of field-based or methods courses. In bi-directional partnerships all aspects of clinical experiences are negotiated with schools and the EPP. For example, effective programs work collaboratively to select and develop mentor teachers who have a command of effective instruction and can articulate what they are doing and why they are doing it.

In a mutually beneficial partnership clinical experiences are designed to support more than just the candidate or to provide extra classroom support for a teacher. The experience can become a system of simultaneous growth and renewal for the teacher candidate-mentor teacher-university supervisor team as they collaborate; and all are learning and leading as they work on behalf of students. Only when preparation programs purposefully engage with schools will their clinical preparation become truly robust in ways that support the development of candidates' skills as related to the needs of schools and school districts.

Indicator P.4.2. Sequence School-Based Experiences: *An effective mathematics teacher preparation program supports teacher candidates' engagement in increasingly comprehensive acts of teaching by providing coherent and developmentally appropriate clinical experiences.*

The quality of school-based experiences is at least as important as the quantity of time in schools. Observations are ineffective for novices that have not been provided with a critical lens from which to gain insights into teaching. Scaffolded learning experiences of teacher candidates support movement toward classroom independence. Therefore, an effective mathematics teacher preparation program develops clear trajectories of school-based learning experiences that focus on increasingly complex teaching practices (e.g., managing student mathematical discourse moving from individual to small group and then from small group to large group) and on learning through examination of student thinking and instructional practice (Smith, 2001). Teacher candidates must have multiple opportunities to practice the many skills that define them as well-prepared beginners (see Chapter 2).

An effective mathematics teacher preparation program includes opportunities for teacher candidates to engage in early clinical experiences to begin to shift their lens from that of a student to that of a teacher, and to gain insights into what grade level(s) they would like to teach. In such early experiences, effective programs provide strategies for helping the candidates to develop their mathematics-teaching lens. For example, candidates may be given assignments to carry out in their field placement to reflect on how teacher-student interactions engage a student in productive struggle, observe or interview a student to understand which mathematical representations the student understands and uses to solve problems, or assist the classroom teacher during individual or small group work documenting the different ways students are reasoning about the task. These early experiences should help potential teachers determine if they want to become teachers, begin to focus on students' mathematical thinking, and introduce basic ideas about effective instruction.

As teacher candidates complete their mathematics methods course(s), their clinical experiences must provide varied and extensive opportunities to connect what they are learning in their coursework to authentic classrooms. For example, during a middle school methods course, prospective teachers may solve a proportional reasoning task and consider different ways students may reason about the task, and then implement the same task with middle school students, returning to the university classroom to debrief about what they learned about proportional thinking, students, and their teaching. Importantly, candidates need multiple opportunities to teach and/or co-teach lessons, including opportunities to analyze student work and reflect on the effectiveness of teaching and classroom management in supporting the mathematics learning of each student in the group or class.

Student teaching or internship experiences (where over the duration of the placement candidates take on the full responsibilities of the classroom) must continue to include a range of activities and assessments that engage the candidate in planning, teaching, assessing, and reflecting on mathematics teaching, at an increasingly sophisticated level in collaboration with the mentor teacher. Internship experiences must have high expectations for candidates to teach mathematics effectively. This includes a focus on culturally responsive instruction, in particular, identifying teaching practices that support or inhibit their students' learning. This requires significant feedback from mentors that are themselves experienced and skilled at effective teaching mathematics. Also, internships must provide opportunities for candidates to engage in important aspects of schooling beyond teaching, such as record-keeping, interacting with parents, and analyzing student data.

Effective teacher preparation programs may vary widely in how they sequence experiences, but they have strategically organized a variety of experiences over time such that their candidates are able to develop the myriad of knowledge, skills, and dispositions to enter the workforce well-prepared to support students' mathematical engagement and understanding. For example, one field experience model posited to foster reflective and student-centered teaching practices is the paired placement model in which two prospective teachers are paired with a single mentor teacher. The mentor teacher

provides purposeful coaching and mentoring; and the two preservice teachers offer each other feedback, mentoring, and support (Goodnough et al., 2009; Leatham & Peterson, 2010). Another model found to help teacher candidates gain greater pedagogical content knowledge and knowledge of students is co-teaching (CT). CT and the paired placement model promote the collaboration and communication between teacher candidates and mentor teachers who share a common space in the planning, implementation, and assessment of instruction (Bacharach, Heck, & Dahlberg, 2010).

Indicator P.4.3. Experience Teaching with Diverse Learners: *An effective mathematics teacher preparation program provides clinical experiences that prepare teacher candidates to teach mathematics to a range of students, in a variety of contexts, and across the grades and content ranges for which they will be certified.*

Teachers of mathematics must be prepared to address the academic, socio-emotional, and cultural needs of the diverse students they serve. Field experiences in diverse settings can change preservice teachers' perspectives on cultural diversity and increase teacher self-efficacy and retention (Castro, 2010; Conaway et al., 2007). An effective program ensures that emerging teachers of mathematics have the opportunity to work with diverse students. Effective programs also insure that emerging teachers have significant experiences at the grades and courses for which the candidate is being certified. For example, a program that certifies teachers grades 7 -12 provides opportunities for candidates to teach diverse learners at both the middle and high school levels.

Placing teacher candidates in a diverse setting is not enough to counteract stereotypical thinking, especially if the experiences do not include opportunities for critical reflection (Bell, Horn, & Roxas, 2007; Garmon, 2004). Effective mathematics teacher preparation programs, therefore, must provide opportunities for candidates to work with a range of students in settings where the mentors and supervisors are able to model and support inclusive and culturally-responsive mathematics instruction and to provide pragmatic strategies for having high expectations for all students (see 3.4.4). Some institutions are located in less diverse geographical locations. It is still critical that their mathematics teacher candidates have opportunities to engage in experiences with diverse learners through other authentic experiences. For example, content-focused pen pal exchanges and the use of video, and live-stream from classrooms can increase opportunities to address implicit bias and develop more complex understandings of working in culturally and economically diverse settings.

Schools vary greatly in their characteristics, such as the extent to which the mathematics content is integrated, the amount of leveling of students into courses, the philosophy on how mathematics is learned, and the instructional materials (e.g. textbooks, online curriculum opportunities) that are used. Despite research that tracking or ability grouping can inhibit learning and cause inequities within the system, NAEP data indicate that an increasing number of schools are using these practices (Loveless, 2013). Effective programs ensure that candidates have opportunities to teach student groups or classes that are considered 'below grade-level', 'at grade-level', and 'above grade-level' and to reflect on the way in which these school structures impact students' opportunities to learn.

Indicator P.4.4. Recruit and Support Qualified Mentor Teachers and Teacher Preparation Supervisors. *Effective mathematics teacher preparation programs ensure that teacher candidates have mentor teachers and supervisors who are able to effectively use clinical settings to support candidates in teaching mathematics well and providing equitable support to all students.*

Clinical experiences are crucial in supporting the development of beginning teachers who can skillfully do the work of mathematics teaching. When done well, clinical experiences provide candidates with scaffolded opportunities to develop skill with teaching practices, insight into mathematics content and into students as learners of that content, and professional orientations and commitments. The quality of these learning opportunities hinges on the support of mentors and teacher preparation supervisors who ensure that candidates are not simply having clinical experiences – but rather actively leveraging those experiences to learn key knowledge, skills, and dispositions (Grossman, 2010; Boyd et al., 2009). In effective programs, candidates have regular opportunities for critical reflection on teaching and learning, as well as on the impact of school structures on student learning.

Effective mentor teachers know mathematics, model effective mathematics teaching practices, and demonstrate professional commitment to the learning of all students. Similarly, supervisors of mathematics candidates have had substantive and successful experience teaching mathematics. They know the mathematics and pedagogical practices that are essential for a well-prepared beginning teacher of mathematics. Further, they know how to utilize the affordances of clinical contexts to support teacher learning. Mentors are willing to open up their own teaching as a site for the beginning teacher to learn. They co-teach to provide focused support during instruction and regularly provide constructive feedback. Supervisors have command of teacher preparation practices that support the integrated attention to students, content, and teaching practices necessary for skilled engagement in teaching and learning contexts (Lampert et al., 2013).

Effective mentors and supervisors of mathematics teachers promote understanding of the context within schools, including helping beginning teachers of mathematics realize the role that administrators, parents, and communities play in supporting the mathematics learning of their students. Further, mentors and supervisors have a strong understanding of what it means to advocate for equitable mathematics learning and can communicate this stance in concrete ways to teacher candidates. For example, a mentor might ask a teacher candidate to reflect on how a particular pedagogical move might connect with or impact students' mathematical identities.

Effective mathematics teacher preparation programs provide organizational structures that support effective mathematics teaching. They recognize that strong partnerships between teacher preparation programs and schools enhance the instruction of beginning teachers (Grossman et al., 2011; Zeichner & Gore, 1990) by connecting the goals and substance of what a candidate is learning through their preparation program and the everyday mathematics learning in PK-12 settings. Program mentors, supervisors, and instructors have ongoing opportunities to connect with program colleagues and other mathematics educators to ensure that teacher candidates have a clear and consistent message about mathematics teaching. Programs ensure that those involved with candidates have ongoing professional development in coaching and mentoring that help them develop knowledge, skills, and dispositions that promote productive work in school contexts.

Standard P.5: Recruit and Retain Teacher Candidates

An effective mathematics teacher preparation program attracts, nurtures, and graduates high quality teachers of mathematics who are representative of diverse communities.

A high-quality teacher of mathematics is one who is well prepared to begin teaching mathematics and who is also capable of achieving and exemplifying the knowledge, skills, and dispositions required for meeting the needs of mathematics learners in schools throughout a sustained teaching career. The well-prepared beginning teacher not only has strong mathematics content knowledge, but is also skilled at teaching and disposed to care deeply about all students (Wilson & Cooney, 2002). At the elementary school level, the teacher's profile must include caring about children as well as having a positive disposition toward mathematics learning and teaching (CAEP, 2013). At the secondary school level, the profile must include a passion for mathematics and for teaching as well as a commitment to positively impacting the learning of all students (CAEP, 2013). Explicit effort is required by the teacher preparation program to recruit and retain a potential teacher workforce that reflects the diverse communities they will serve. Teacher education programs that address this standard, its indicators below, and the other standards in this document are more likely to produce high quality teacher candidates who will remain in the profession and have an impact on student learning over a career (Darling-Hammond, 2000).

There is a significant difference in racial and ethnic demographics of the population of students in today's mathematics classes and their teachers who are predominantly female, white, and monolingual (US Department of Education, 2013). The lower college enrollment rates of students of color and barriers such as licensure examinations limit efforts to increase the diversity of mathematics teachers (Ahmad & Boser, 2014). Even with low enrollments, preparation programs must actively recruit and support a diverse teacher candidate pool. Recruit efforts must also include providing support for candidates. For example, high quality teacher candidates must have a profile that not only includes evidence of or potential for excellent academic achievement, but must also demonstrate promise for broader qualities or competencies. Exemplary achievements related to working with and meeting the needs of every child learning mathematics particularly those who may have been traditionally discriminated against, excluded, or marginalized are as important as high performance on traditional measures of academic achievement.

Indicator P.5.1. Recruit Strong Candidates: *An effective mathematics teacher preparation program uses a strategic process for recruiting candidates who are capable of successfully addressing the standards in this document.*

The process of recruiting future teachers of mathematics involves a multiple step approach that includes informing potential candidates about different options and opportunities in teaching, providing experiences that allow them to work with learners of mathematics, and then assisting them in applying for admission to and seeking financial support for completing a teacher education program. Effective mathematics teacher preparation programs must carefully and strategically manage and involve staff and faculty in all aspects of the recruitment process (Dickey, 2016).

For example, the Mathematics Teacher Education Partnership Research Action Cluster (Ranta & Dickey, 2015) on recruitment has found that strategies specific to attracting high school and college students to programs leading to mathematics certification or licensure include:

- Offering field experiences in school mathematics settings with exemplary teachers
- Providing scholarships targeted to high-need programs
- Promoting the need for secondary mathematics teachers that exceeds the need for elementary teachers as well as for secondary level English/Language Art or Social Studies teachers
- Highlighting the integrated and active learning curriculum intended for elementary and middle grades learners
- Building a connection to the unique emotional and cognitive needs of adolescent learners
- Providing career counseling to liberal arts majors about major changes and certification options specific to teaching mathematics.

Mathematics teacher preparation programs assess candidates' qualifications for admission using multiple measures that include both cognitive and dispositional factors. This process should uncover their passion for and commitment to mathematics as a discipline and an essential component of an effective citizenry, as well as their stances on embracing opportunities to learn, their commitment to helping students grow, and their commitment to equitable teaching. Consistent with CAEP (2013) Standard 2, cognitive measures that include grades in mathematics content courses or standardized mathematics test scores provide valuable information, but are not fully sufficient. Identifying dispositions of applicants should be part of an admission interview or essay that the mathematics teacher preparation program uses to inform both admissions and program planning decisions. For all candidates, prompts that seek to uncover a implicit biases or deficit views of diverse children and families as well as dispositions to avoid mathematics such as "I was never good at math" or "I prefer not to teach math" should be included to allow counseling or a program decision that will improve teacher candidates' attitudes. Additionally, prompts should be used that address a commitment to the needs of all learners and that elicit a commitment to and passion for teaching and mathematical habits of mind.

Indicator P.5.2. Address Diverse Community Needs: *An effective mathematics teacher preparation program actively seeks to address the diverse needs of its communities by recruiting future teachers of mathematics using a variety of strategies that include outreach in schools, community colleges, and within the institution.*

The diverse needs among different communities and locales vary, but mathematics teacher preparation programs should aspire to produce new teachers whose demographics mirror those of the community they serve (Ahmad & Boser, 2014; Goldhaber & Hansen, 2010). To actively recruit to meet local diversity needs and to address the critical shortage of secondary school mathematics teachers, the mathematics teacher preparation program should work within middle and high schools particularly those with clubs or future teacher groups tied to education careers to build interest in the profession of teaching mathematics through activities like peer tutoring or dual enrollment courses. These early attempts to reach potential candidates prior to college help plant the seed of being a mathematics teacher as a member of a rewarding profession. Reaching out to community colleges as well as to current STEM majors at the program's institution also provides a means of diversifying the candidate pool. In addition, actively recruiting paraprofessional educators already working in schools, parent or other volunteers, and camp counselors or coaches working in informal educational settings can help diversify a program's applicant pool.

Indicator P.5.3. Provide Experiences and Support Structures: *An effective mathematics teacher preparation program provides experiences and support structures to ensure the short- and long-term success of their teacher candidates as well as to enhance the promise demonstrated at admission to the program.*

The mathematics teacher preparation program provides appropriate support structures through candidate counseling and advisement, tutoring support in mathematics content, and assistance with Praxis or other related (including state-required) summative or performance assessment preparation (Seymour & Hewitt, 1997). Early and frequent field experiences in a variety of settings and targeted experiences across grade levels ensure greater possibilities for success in teaching both in the short- and long-term (Clift & Brady, 2006). For early childhood, elementary, and special education teacher candidates, the mathematics teacher preparation program provides appropriate placements at varying grade levels to assist in the growth of the candidates' mathematical knowledge for teaching. For secondary programs, the mathematics teacher preparation program systematically diversifies placements to ensure candidates gain experience in diverse courses, grade levels, and settings (CAEP, 2013). At any certification level, up to 200 hours of field experiences in multiple classrooms and environments prior to student teaching should be required (CAEP, 2013). Even when programs' clinical experiences are shorter in duration, as they might be at the post-baccalaureate or graduate levels, collecting information about candidates' past experiences with children and schools allows for targeting placements that complement and diversify the collection of experiences.

Indicator P.5.4. Assess Recruitment and Retention Data: *An effective mathematics teacher preparation program includes an assessment system that critically examines recruitment and retention practices in a manner that improves results based on data review and analysis by program faculty and partners/stakeholders.*

Consistent with Standard 5 of the Council for the Accreditation of Educator Preparation (CAEP, 2013), the mathematics teacher preparation program assessment system includes expectations with measurable outcomes tied to effective recruitment and retention efforts. State-accredited or any mathematics teacher education program should also employ a comparable assessment system. Mathematics teacher preparation programs should aspire to assess the quality of their outcomes by setting goals for recruiting new mathematics teachers candidates, systematically gathering data and analyzing recruitment practices (e.g., impact on admissions from recruiting at high schools, community colleges, or other STEM majors), and conducting case studies of both successful and unsuccessful candidates to gain insights on factors that influence program completion of a well-prepared beginning teacher. The mathematics teacher preparation program should gather and review data on whether Praxis or other required candidate assessment test preparation programs provide the intended support and whether retention programs, either those organized by the program or by the graduate's employer, impact success of the beginning teacher. Similarly, the program should maintain data and examine results from other state or national teacher performance assessment systems to ensure validity and examine how the data support or impact recruitment and retention as well as how the data might help improve the quality of the program.

Educator preparation programs must also monitor the early career progress of their graduates to inform program improvement as well as to contribute the retention of teachers. Using social media can allow

programs to maintain contact with graduates and create communities that provide peer and institutional support after program completion.

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CHAPTER 4. ELABORATIONS OF THE STANDARDS FOR THE PREPARATION OF EARLY CHILDHOOD TEACHERS OF MATHEMATICS

This chapter puts forth elaborations and examples of the standards in Chapter 2, describing the knowledge, skills, dispositions, and actions that well-prepared beginning Early Childhood mathematics teachers need to develop, followed by elaborations and examples of the standards in Chapter 3, describing what pre-K to grade 2 level preservice programs need to do to ensure the effective preparation of their candidates.

Although young children are ready and eager to learn, many early childhood teachers are not as eager nor prepared to engage children in rich experiences in domains other than literacy (Institute of Medicine [IOM] & National Research Council [NRC], 2015; NRC, 2001b; NRC, 2007). Early childhood educators teach children from birth to age 8 years, an especially critical developmental period of learning for mathematics. These early years form the cognitive foundation of mathematical thinking. Preschool children's knowledge of mathematics predicts their later school success in elementary school (Duncan et al., 2007). Further, early performance in mathematics predicts later reading achievement, as well as early reading skills (Lerkkanen, Rasku-Puttonen, Aunola, & Nurmi, 2005). The quantitative, spatial, and logical reasoning competencies of mathematics establish an early cognitive foundation for thinking and learning across subjects.

Given the importance of mathematics to academic success (National Mathematics Advisory Panel, 2008), all children need to develop a robust knowledge of mathematics in their earliest years. The development of this knowledge among children, as well as the formation of their beliefs and dispositions toward mathematics, is dependent upon and related to the capabilities and dispositions of their teachers (Tsamir & Tirosh, 2009). Given the limitations in the present preparation of the early childhood workforce in the domain of mathematics education (IOM & NRC, 2015), it is imperative that teacher preparation programs attend to the standards set forth in this document. Effective teacher preparation programs at the early childhood level include focused, sustained, and substantial attention to developing prospective teachers' deep understanding of early mathematics, knowledge of children's developmental learning trajectories for mathematics, and high-leverage pedagogical skills and practices for teaching mathematics.

Building on the standards presented in Chapter 2, this chapter elaborates on the knowledge, skills, dispositions, and actions of well-prepared early childhood teachers of mathematics. Effective teacher preparation programs at the early childhood level not only meet the expectations of all the program and candidate standards, they demonstrate attention to the elaborations listed in Table 4.1.

Table 4.1. Elaborations of Candidate Standards for Early Childhood Teachers of Mathematics

Part 1: Candidate Knowledge, Skills, and Dispositions

Deep Understanding of Early Mathematics

Well-prepared beginning teachers at the early childhood level have a deep understanding of the mathematical concepts and processes important in early learning, as well as knowledge beyond what they will teach. [Elaboration of C.1.1]

Positive Dispositions toward Mathematics

Well-prepared beginning teachers at the early childhood level demonstrate positive dispositions toward mathematics as a discipline and toward the teaching and learning of mathematics. [Elaboration of C.1.3]

Mathematics Learning Trajectories: Paths for Excellence and Equity

Well-prepared beginning teachers at the early childhood level understand learning trajectories for key mathematical topics, including how these learning trajectories connect to foundational knowledge, as well as curriculum and assessment frameworks. [Elaboration of C.1.4]

Tools, Tasks, and Talk as Essential Pedagogies for Meaningful Mathematics

Well-prepared beginning teachers at the early childhood level intentionally plan for and use tools, tasks, and talk as pedagogies for young children's engagement in meaningful mathematics. [Elaboration of C.2.2 & C.2.3]

Understanding Young Children's Mathematical Thinking Informs Teaching

Well-prepared beginning teachers at the early childhood level elicit and analyze young children's mathematical thinking to inform classroom interactions and instructional decisions. [Elaboration of C.2.3]

Collaboration with Families Enhances Children's Mathematical Development

Well-prepared beginning teachers at the early childhood level collaborate with families in a mutually respectful, reciprocal way, to enhance and connect children's in-school and out-of-school mathematical development. [Elaboration of C.2.5]

Seeing Mathematics through Children's Eyes

Well-prepared beginning teachers at the early childhood level are conversant in the developmental progressions that are the core component of learning trajectories, thus seeing mathematical situations through children's eyes. [Elaboration of C.3.1]

Creating Positive Early Childhood Learning Environments

Well-prepared beginning teachers at the early childhood level create mathematical learning environments, characterized by exploration, reasoning, and problem solving, that draws upon children's mathematical, cultural, and linguistic strengths thereby developing conceptual understanding and positive mathematical identities. [Elaboration of C.4.2 & C.4.3]

Advocating for Every Young Child’s Access to High-quality Mathematics Learning Experiences

Well-prepared beginning teachers at the early childhood level advocate for every child to have access to high-quality mathematical learning experiences in preschool and primary settings.

[Elaboration of C.4.1, C.4.4, & C.4.5]

Part 2: Program Characteristics and Qualities

Providing Diverse Clinical Settings: Preschool and Primary

Programs preparing teachers of young children include opportunities to learn in both preschool and primary settings. [Elaboration of P.4]

Part 1: Elaborations of the Knowledge, Skills and Dispositions Needed by Well-prepared Beginning Early Childhood Mathematics Teachers

This section provides additional detail, commentary, and examples of the knowledge, skills, and dispositions well-prepared Early Childhood mathematics teachers should have, organized by the general standards described in Chapter 2.

Standard C.1. Knowledge of Mathematics for Teaching

Effective teachers have a deep understanding of the mathematics they are expected to teach and exhibit positive dispositions toward both mathematics teaching and learning. Such understanding and dispositions are particularly critical for early childhood teachers, as they develop the foundation of mathematical understanding, beliefs, and attitudes among young learners that start students on their mathematical journey. Therefore, we have a critical elaboration of this standard for the preparation of early childhood teachers of mathematics.

Deep Understanding of Early Mathematics

Well-prepared beginning teachers at the early childhood level have a deep understanding of the mathematical concepts and processes important in early learning, as well as knowledge beyond what they will teach. [Elaboration of C.1.1]

Although it may seem obvious that all teachers should understand the mathematics they are to teach, many have not had experiences in the U.S. educational system to understand key conceptual understandings of the major topics of early mathematics (Ma, 1999; NRC, 2009). Effective teachers at the early childhood level hold deep conceptual understanding of the mathematics they teach, as well as knowledge of how these foundational mathematical ideas connect to subsequent learning on the mathematical horizon.

Well-prepared beginning teachers of mathematics possess appropriate mathematical knowledge of and skill in mathematics needed for teaching. They engage in appropriate mathematical practices and support their students in doing the same. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students’ mathematical productions.

Indicators include:

- C.1.1. Core content knowledge
- C.1.2. Mathematical practices and processes
- C.1.3. Mathematical dispositions
- C.1.4. Analyzing the mathematical content of curriculum
- C.1.5. Mathematical knowledge to analyze mathematical thinking
- C.1.6. Using mathematical tools and technology

The depth of the mathematics in the early years is often misunderstood (Lee & Ginsburg, 2009). The “mathematical ideas that are suitable for preschool and the early grades reveal a surprising intricacy and complexity when they are examined in depth. At the deepest levels, they form the foundations of mathematics that have been studied extensively by mathematicians over centuries ... and remain a current research topic in mathematics” (NRC, 2009, p. 21). As an example, teachers need to understand how counting relates to place value, in that only 10 digits are needed to write any counting number by creating larger and larger units, which are the values of places in a written numeral, by taking the value of each place to be equal to 10 of the place to its right (e.g., the “1” in 125 is equal to 10 tens). In this way, every counting number can be expressed in a unique way as a numeral made of a string of digits. These ideas connect the study of counting and place value and are essential in the development of arithmetic, such as understanding that $63 + 10$ is 73 without having to count by ones. Full appreciation of the multiplicative relationship in place value (taking the value of each place to be 10 *times* the value of the previous place to its right) developed over time, is critical for further mathematics learning, such as understanding decimal numbers.

Programs must prepare prospective teachers with a broad and deep understanding of fundamental mathematics. The most important is the domain of number and the related concepts of quantity and relative quantity, counting, and arithmetical operations. Also critical are the domains of geometry and measurement, through which people mentally structure the spaces and objects around them. Connections and coherence among mathematical ideas are enriched as candidates apply number concepts and processes to these spatial structures. In addition, these domains provide a rich context to further develop the ability to reason mathematically.

In Figure 4.1, we highlight the important concepts that a well-prepared beginning teacher of mathematics must know in order to be able to support learners in grades PK through two. These concepts, identified in *The Mathematical Education of Teachers II* (CBMS, 2012), focus on counting and cardinality, operations and algebraic thinking, number and operations in base ten, measurement and data, and geometry. The mathematical ideas at the early childhood level form the subtle and complex foundation of school mathematics. Prospective early childhood teachers and the programs that prepare them should not dismiss these foundational ideas as simple, but rather treat them with the mathematical respect that a careful and sustained examination affords. These ideas are closely connected to their grades 3-5 successors (as addressed in chapter 5), and programs preparing early childhood teachers should ensure that they offer prospective teachers the opportunity to examine the full spectrum of mathematical connections across the preschool through grade 5 span.

This examination of foundational mathematics must include more than learning appropriate content. Prospective early childhood teachers should continually engage in the mathematical practices and processes of reasoning, sense-making, and problem solving. They should look for opportunities to use mathematics to analyze real-world situations.

Figure 4.1. Connections to the CBMS (2012) Report on *The Mathematical Education of Teachers II* for Early Childhood Teacher Preparation Programs

Counting and Cardinality

- The intricacy of learning to count, including the distinction between counting as a list of numbers in order and counting to determine a number of objects.

Operations and Algebraic Thinking

- The different types of problems solved by addition, subtraction, multiplication, and division, and meanings of the operations illustrated by these problem types.
- Teaching–learning paths for single-digit addition and associated subtraction, including the use of properties of operations.
- The foundations of algebra in elementary mathematics, including understanding the equal sign as meaning “the same amount as” rather than a “calculate the answer” symbol.

Number and Operations in Base Ten

- How the base-ten place value system relies on repeated bundling in groups of ten and how to use objects, drawings, layered place value cards, and numerical expressions to help reveal base-ten structure. Developing progressively sophisticated understandings of base-ten structure as indicated by these expressions:

$$\begin{aligned} 357 &= 300 + 50 + 7 \\ &= 3 \times 100 + 5 \times 10 + 7 \times 1 \\ &= 3 \times (10 \times 10) + 5 \times 10 + 7 \times 1 \\ &= 3 \times 10^2 + 5 \times 10^1 + 7 \times 10^0. \end{aligned}$$

- How efficient base-ten computation methods for addition and subtraction rely on decomposing numbers represented in base ten according to the base-ten units represented by their digits and applying (often informally) properties of operations, including the commutative and associative properties of addition, to decompose the calculation into parts. How to use mathematical drawings or manipulative materials to reveal, discuss, and explain the rationale behind computation methods.

Measurement and Data

- The general principles of measurement, the process of iterations, and the central role of units: that measurement requires a choice of measurable attribute, that measurement is comparison with a unit and how the size of a unit affects measurements, and the iteration, additivity, and invariance used in determining measurements.
- How the number line connects measurement with number through length.
- Using data displays to ask and answer questions about data, and analyzing data with attention to the shape and spread.

Geometry

- Understanding geometric concepts of angle, parallel, and perpendicular, and using them in describing and defining shapes; describing and reasoning about spatial locations (including the coordinate plane).
- Classifying shapes into categories and reasoning to explain relationships among the categories.

Positive Dispositions toward Mathematics

Well-prepared beginning teachers at the early childhood level demonstrate positive dispositions toward mathematics as a discipline and toward the teaching and learning of mathematics.

[Elaboration of C.1.3]

It is particularly important that positive dispositions toward mathematics are cultivated during the mathematical preparation of early childhood teachers, because teachers' beliefs and affect toward mathematics influence what students come to believe and feel toward mathematics (Tsamir & Tirosh, 2009; White, Perry, Way, & Southwell, 2006), and teachers' beliefs influence the opportunities they provide for students to engage in significant mathematical thinking (Staub & Stern, 2002). Children's early experiences with mathematics form the foundation for their future success as mathematics learners. That foundation includes not only the development of mathematical knowledge, but also the establishment of productive dispositions toward mathematics. Productive dispositions are defined as the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2001a, p. 116). As teachers make decisions about the mathematical tasks, tools, and discourse within the learning environment, they influence the mathematics content knowledge that children develop, as well as children's identities as mathematics learners (Aguirre, Mayfield-Ingram, & Martin, 2013).

The mathematical preparation of teachers plays a critical role in developing more productive beliefs and attitudes toward mathematics (Philipp, 2007; Swars, Smith, Smith, & Hart, 2009). Prospective elementary teachers' attitudes toward mathematics are impacted at two key points (Jong & Hodges, 2015). First, positive shifts in attitudes toward mathematics occur as a result of mathematics methods coursework and, second, further positive shifts occur as a result of clinical experiences. Throughout the mathematical preparation of early childhood teachers, increased attention must be given to the development of positive dispositions toward mathematics as a necessary requirement for the teaching of more rigorous expectations for mathematics, including that of our youngest learners in preschool and primary settings.

Vignette: Examining Memories of Learning Mathematics

Each semester on the first day of my mathematics methods course, I ask my prospective teachers to reflect on their memories of learning mathematics in elementary school, middle school, and high school. They are given about ten minutes to write about their own memories of their mathematical experiences (e.g., people, activities, topics, expectations for learning). I encourage them to put on paper not only some specific examples of what they remember doing, but to also describe how they felt in those mathematical situations.

Needless to say, the memories and stories are not all positive. Next they form small groups and share their memories. After this, we discuss and chart common themes as a whole group, and surface both positive and negative memories. As memories are shared, I often ask them to comment on feelings about themselves as a learner of mathematics and about the expectations for understanding or making sense of the mathematics in each of those situations. Unfortunately, over the years, far too few of the prospective early childhood teachers share positive memories. Mathematics was, for many, just something they had to do, and it was not a favorite subject. For others, those reflections evoke bad memories, generate tears, and raise anxieties. Whether the memories were positive or negative, a

common theme surfaces that mathematics was for most, something to memorize and not something to understand or enjoy.

To close the session, I have the prospective teachers reflect and write once again. This time, I ask them to put on paper, how they would like to be remembered as a teacher of mathematics. As the instructor, I then collect, read, and respond to their writing. This initial experience serves as a backdrop for our work together throughout the semester as each of the prospective teachers delves more deeply into what it means to be an effective teacher of mathematics at the early childhood level.

Related Resources

Guillaume, A. M., & Kirtman, L. (2005). Learning lessons about lessons: Memories of mathematics instruction. *Teaching Children Mathematics*, 11(6), 302.

McCulloch, A. W., Marshall, P. L., DeCuir-Gunby, J. T., & Caldwell, T. S. (2013). Math autobiographies: A window into teachers' identities as mathematics learners. *School Science and Mathematics*, 113(8), 380-389.

Mathematics Learning Trajectories: Paths for Excellence and Equity

Well-prepared beginning teachers at the early childhood level understand learning trajectories for key mathematical topics, including how these learning trajectories connect to foundational knowledge, as well as curriculum and assessment frameworks. [Elaboration of C.1.4]

Learning trajectories are effective frameworks to use as a way to develop foundations of early mathematics. They are especially important in early childhood for five reasons (Clements & Sarama, 2014). First, children's cognitive development influences how they think and what they can learn about mathematics arguably more in these early years than at any other age. Often the variance among children on a variety of cognitive factors is also particularly wide at this level. Second, contexts for teaching (e.g., whole group, learning centers, small groups, individual interactions, informal settings) are varied in early childhood and difficult to coordinate and use for helping children learn mathematics without a good understanding of learning trajectories. Third, the younger the children the more important it is that teachers use children's thinking and prior knowledge as starting points. Fourth, much is known about learning trajectories in *early* mathematics, more than any other age range (see Figure 4.2 for a listing of current research-based learning trajectories). Fifth, research and developmental work indicates that learning trajectories are effective guides for informing instructional approaches that support young children's learning of mathematics. They also help early childhood educators respect children's developmental processes and constraints, as well as their potential for thinking about and understanding mathematical ideas.

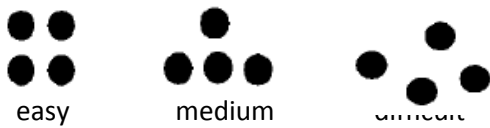

Figure 4.2. Research-based Developmental Learning Trajectories

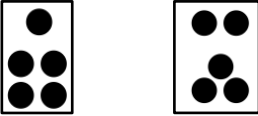
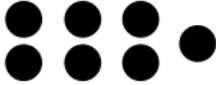
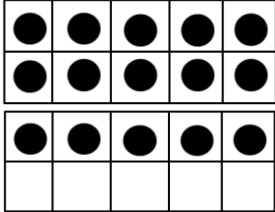
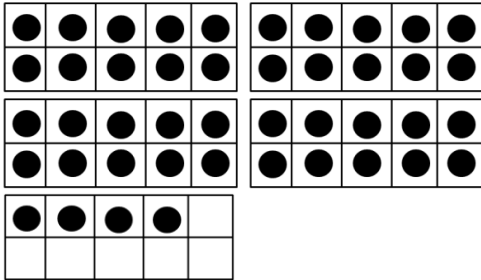
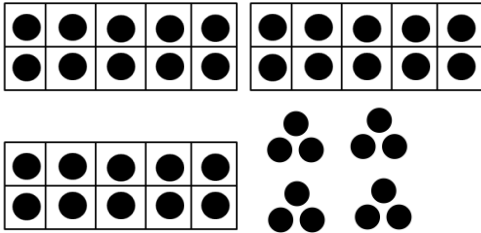
- Recognition of Number and Subitizing
- Verbal and Object Counting
- Comparing, Ordering, and Estimating Numbers
- Early Addition and Subtraction
- Composing Number and Multidigit Addition and Subtraction
- Early Multiplication and Division and Fractions
- Spatial Thinking
- Shapes
- Composition of 3D Shapes
- Composition and Decomposition of 2D Shapes
- Length Measurement
- Area Measurement
- Volume Measurement
- Angle Measurement

Each learning trajectory has three components, a mathematical *goal*, a *developmental progression*, and *instructional strategies* (Sarama & Clements, 2009). To develop a particular mathematical understanding (the goal), children construct each level of thinking and reasoning sequentially (the developmental progression), if provided with appropriate teaching approaches and tasks (the instructional strategies). Prospective early childhood teachers need to learn that these components are intimately and intrinsically interconnected in high-quality instruction. That is, the key to true understanding and successful use of learning trajectories lies not just in understanding each component, but in understanding how the components work together and must be used in concert to engage and support young children's learning and thinking about mathematics. This is a demanding expectation, and thus we recommend that prospective teachers study in depth at least three research-based developmental learning trajectories. For example, candidates might study learning trajectories for subitizing (shown in Figure 4.3), early adding and subtracting, and length measurement. This in-depth study would include instructional planning and application of all three integrated components for each learning trajectory examined, thus serving as a model for utilization of other development trajectories. In addition, prospective teachers should become familiar with other developmental learning trajectories for early mathematics, including how to find resources to extend their own knowledge and support eventual implementation in their own classrooms.

Figure 4.3. A Learning Trajectory for Recognition of Number and Subitizing

(Clements & Sarama, 2014, p. 17-20; copyright 2014 from *Learning and teaching early math: The learning trajectories approach* by Clements & Sarama. Reproduced by permission of Taylor and Francis Group, LLC, a division of Informa plc.)

Age (years)	Developmental Progression	Sample Instructional Tasks
0-1	Pre-Explicit Number. Within the first year, dishabituates to number, but does not have explicit, intentional knowledge of number. For infants, this is first collections of rigid objects one.	<i>Noticing Collections.</i> Provide a rich sensory environment, use of words such as “more” and actions of adding objects, with direct attention to comparisons.
1-2	Small Collection Namer. Names groups of 1 to 2, sometimes 3. Shown a pair of shoes, says, “Two shoes.”	<i>Board Games–Small Numbers:</i> Play board games with a special die (number cube) or spinner that shows only 1, 2, and 3 dots (then add 0 to it).
3	Maker of Small Collections. Nonverbally makes a small collection (no more than 4, usually 1–3) with the same number as another collection (via mental model; i.e., not necessarily by matching). Might also be verbal. When shown a collection of 3, makes another collection of 3.	<i>Get the Number:</i> Ask children to get the right number of crackers or some other item for a small number of children.
4	Perceptual Subitizer to 4. Instantly recognizes collections up to 4 briefly shown and verbally names the number of items. When shown 4 objects briefly, says “four.”	<i>Snapshots:</i> Play “Snapshots” with collections of 1 to 4 objects, arranged in line or other simple arrangement, asking children to respond verbally with the number name. Start with the smaller quantities and easier arrangements, moving to those of moderate difficulty only as children are fully competent and confident.  easy medium difficult
5	Perceptual Subitizer to 5. Instantly recognizes briefly shown collections up to 5 and verbally names the number of items. Recognizes and uses spatial and numeric structures beyond the situations in which they were already experienced. Shown 5 objects briefly, says “five.”	<i>Snapshots:</i> Play “Snapshots” with dot cards, starting with easy arrangements, moving to more difficult arrangements as children are able.  easy medium difficult

5	Conceptual Subitizer to 5. Verbally labels all arrangements to about 5, when shown only briefly. “5! Why? I saw 3 and 2 and so I said five.”	<i>Snapshots.</i> Use different arrangements of the various modifications of “Snapshots” to develop conceptual subitizing and ideas about addition and subtraction. The goal is to encourage students to see the addends and the sum. 
5	Conceptual Subitizer to 10. Verbally labels most briefly shown arrangements to 6, then up to 10, using groups. “In my mind, I made two groups of 3 and one more, so 7.”	<i>Snapshots.</i> Play “Snapshots” with larger quantities to develop ideas about addition and subtraction. 
6	Conceptual Subitizer to 20. Verbally labels structured arrangements up to 20, shown only briefly, using groups. “I saw three fives, so 5, 10, 15.”	<i>Ten Frame Addition Snapshots:</i> Briefly show two ten frames to help children visualize addition combinations. 
7	Conceptual Subitizer with Place Value and Skip Counting. Verbally labels structured arrangements, shown only briefly, using groups, skip counting, and place value. “I saw groups of tens and twos, so 10, 20, 30, 40, 42, 44, . . . 44!”	<i>Ten Frame Addition Snapshots.</i> Briefly show several ten frames. 
8	Conceptual Subitizer with Place Value and Multiplication. Verbally labels structured arrangements shown only briefly, using groups, multiplication, and place value. “I saw groups of tens and threes, so I thought, 3 tens is 30 and 4 threes is 12, so 42 in all.”	<i>Snapshots with Dots.</i> Play “Snapshots” with structured groups that support the use of increasingly sophisticated mental strategies and operations. 

Standard C.2. Knowledge and Pedagogical Practices for Teaching Mathematics

Teaching mathematics is complex. It entails not only knowing the mathematics, but knowing how to design and implement rich mathematics learning experiences that advance students' mathematical knowledge and proficiencies. Effective teachers are skilled in their use of high-leverage mathematics teaching practices and use those pedagogical practices to guide both their preparation and enactment of mathematics lessons. The development of these content-focused skills and abilities, that is teaching practices specific to mathematics, should form the core of work in the preparation of early childhood teachers of mathematics. Therefore, we have a critical elaboration of this standard focused on the knowledge and pedagogical practices specific to the early childhood level.

Well-prepared beginning teachers of mathematics have a foundation of pedagogical content knowledge, effective and equitable mathematics teaching practices, and a positive and productive disposition toward teaching mathematics to support students' sense-making, understanding, and reasoning.

Indicators include:

- C.2.1. Promoting equitable teaching
- C.2.2. Planning for effective instruction
- C.2.3. Implementing effective instruction
- C.2.4. Analyzing teaching practice
- C.2.5. Enhancing teaching through collaboration with colleagues, families, and community members

Tools, Tasks, and Talk as Essential Pedagogies for Meaningful Mathematics

Well-prepared beginning teachers at the early childhood level intentionally plan for and use tools, tasks, and talk as pedagogies for young children's engagement in meaningful mathematics.

[Elaboration of 2.2.2 & C.2.3]

Effective teaching entails meeting children where they are mathematically on a learning trajectory and using instructional pedagogies that utilize tools, tasks, and talk to support advancement in children's mathematical understanding and skills. Prospective teachers in effective teacher preparation programs learn to ask and answer fundamental questions: Is this child or group of children on the learning trajectory as expected for their age and grade? If not, where are they on the trajectory? Where do they need to move next mathematically? How can I, as their teacher, provide instructional experiences that helps them progress in their understanding and use of mathematics? What instructional tasks, tools, and activities might be most beneficial to support their learning? How can I engage these young learners in mathematical conversations and discourse that helps them connect their experiences and informal language with the world of mathematics? What questions should I be asking them to draw out their observations and engage them in mathematical talk?

Well-prepared beginning teachers select tasks purposefully that prompt children to use tools in solving mathematical problems and that support student progress on specific learning trajectories. They know that not all tasks provide the same opportunities for student thinking and learning and that even young children need regular experiences with high-level tasks (Hiebert & Wearne, 1993; Stein, Grover, & Henningsen 1996). They also know that young children are good problem solvers and learn through problem solving experiences (Cai, 2003; Moser & Carpenter, 1982). These teachers see their role as helping students mathematize their world while nurturing understanding of mathematical concepts and relationships and developing language to talk about those emerging observations. They situate mathematical tasks in students' ways of knowing and learning, including attention to students' culture, language, gender, socioeconomic status, cognitive and physical abilities, funds of knowledge, and personal interests. In addition, these teachers know not to rush students toward procedural fluency but

rather that it builds from conceptual understanding through use of informal reasoning strategies in solving problems and that it develops over long periods of time, from months to years (NCTM, 2014; NRC, 2001a).

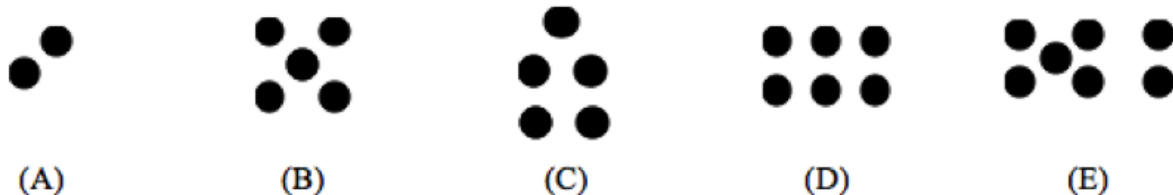
Teachers must learn to see and view mathematics through the eyes of their students, especially in the early childhood years, where students' conceptions can be quite different from those of the teacher. Teachers know that all mathematical ideas are abstract and learners only have access to those ideas through representations (NRC, 2001a). This is especially true among young children that are newly experiencing ways to mathematize their experiences and observations with physical objects, verbal analogies, and drawings, as well as with invented and standard symbolic representations. Thus, well-prepared beginners demonstrate their own representational competence in using physical, visual, verbal, symbolic, and contextual representations appropriate for early mathematics, as well as how these representations are foundational ideas for later mathematics. In addition, they know how to strategically use mathematical tools (e.g., part-whole mats, number bonds, Rekenreks/math racks, ten frames, bead strings, and tape diagrams) to develop and advance children's mathematical understanding and skills.

The work of teaching is complex, regardless of the age of the students. This includes not only attention to tasks and tools for mathematical inquiries, but also attention to involving young learners in meaningful mathematical talk or discourse as they engage in structured mathematical activities, as well as in play (Van Oers, 2010). "Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29). Math talk is particularly important for young children given their limited, but emerging, abilities to write words and use mathematical symbols. Well-prepared beginners know that young children need many opportunities to talk about their mathematical observations and emerging ideas, as well as to listen to and learn from their peers. Thus, prospective teachers must become skillful in noticing and eliciting children's thinking and then engaging them in learner-focused dialogue, or math talk, that links children's informal experiences and words to more formal mathematical ideas, using language appropriate for the particular learners (Rudd, Lambert, Satterwhite, & Zaier, 2008; Whitin & Whitin, 2000).

Vignette: Building from What Children Understand Mathematically

The prospective teachers in a mathematics methods course were asked to read the short narrative shown below and analyze the thinking of two children, Ethan and Morgan.

Ethan and Morgan are both four years old and are enrolled in a preschool program. This is Morgan's second year in the program, whereas Ethan is new this year. One day the teacher checked their subitizing ability through an informal assessment. Working with one child at a time, she flashed each dot patterns shown below for a few seconds and then the child was asked, "How many dots did you see?" If the child hesitated or was unsure, the pattern was shown again or given to the child to examine more closely. Then the child was asked to explain what he/she saw and how he/she determined the total number of dots.



Ethan was able to identify two dots at a glance (Pattern A), but needed to count the other sets of dots by ones by touching each dot with his finger. For Pattern C, he guessed “Eight” and for Pattern E he said, “I call that one eleven.” When he was handed the dot patterns, he counted the dots by ones and was able to recite the number names in the correct order, but often had to re-count the sets as he often lost track of which dots he had counted and which still needed to be counted. He demonstrated a connection between his counting and the cardinality of each set.

Morgan was able to quickly identify the total number of dots in each set without having to count by ones. For Patterns A, B, and D, the familiar dice configurations, she very quickly identified the total number of dots as two, five, and six. When asked, “How do you know it’s five?” for Pattern B, she replied, “Cuz there’s two and two and one in the middle. Five!” For Pattern D, she explained, “Cuz they’re in the right order.” She hesitated for only a moment with Pattern C and then said it was five dots and explained, “Cuz these are four and this is five, but not in the middle” as she pointed to the familiar arrangement of four dots on the bottom of the card and then pointed to the one dot on top.

Working in pairs, the prospective teachers placed each child on the subitizing learning trajectory, and then made instructional suggestions for advancing the learning of each child. This included identifying specific tasks and useful representations, formulating purposeful questions to ask the children, and articulating the mathematical knowledge and reasoning being targeted with each task. Then the whole class reconvened and collectively reached consensus on each child’s learning trajectory placement and next instructional steps.

Understanding Young Children’s Mathematical Thinking Informs Teaching

Well-prepared beginning teachers at the early childhood level elicit and analyze young children’s mathematical thinking to inform classroom interactions and instructional decisions. [Elaboration of C.2.3]

Educational assessments serve a variety of purposes. Sometimes assessments are high-stakes and are summative in nature. Other times, they serve a diagnostic function, as in the identification of children with special needs. Within the daily classroom interactions of the teacher and students, assessment is formative in nature, it guides instruction and gauges whether instruction has been effective or needs modification. Research and expert opinion (Clarke, Clarke, & Roche, 2011) suggest that the primary goal of assessing young children should be to understand children’s thinking and knowledge and to inform ongoing teaching efforts. Performance tasks (meaningful activities that require children to synthesize and apply knowledge and skills by making a response or creating a product) and informal ongoing assessments such as observations, interviews/questioning sequences, paper-and-pencil tasks, computer-based tasks, and digital records (audio and video) are useful and informative ways of

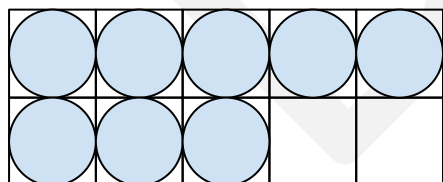
assessing young children’s mathematical learning and should be integrated as appropriate into the early childhood mathematics curriculum.

Performing effective formative assessment supportive of early childhood learning requires that teachers understand developmental progressions of learning trajectories and set goals that are appropriate for both the children in their classroom and responsive to each child’s learning needs. They also need to know *how* to assess children to identify their strengths and current level(s) of understanding and what level they are likely to learn next. In the vignette “Collaborative Sense Making of Children’s Mathematical Thinking,” such a sequence of eliciting and using children’s thinking to inform and adjust instruction takes place. The data about children’s current thinking supports the team as they collaboratively determine how the current thinking connects to the progression and ways they may support movement along the content trajectory and development of the children’s engagement with the mathematical practices.

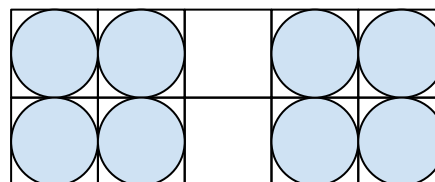
Vignette: Collaborative Sense Making of Children’s Mathematical Thinking

The prospective teachers in a mathematics methods course were asked to conduct a Number Talk in their classroom setting. They engaged in pre-planning the Number Talk by anticipating student thinking and planning for how they might record their ideas using images and/or numbers and math symbols to convey the children’s thinking. Next, they reviewed the plan with their cooperating teacher (and/or university supervisor) who made suggestions prior to implementation and took notes about the children’s thinking during the lesson. After the lesson, they debriefed to consider next instructional steps based on the thinking that emerged.

A teacher candidate in a first grade classroom in February provided her students with “Quick Images” of dots within a tens frame. She was interested in having students conceptualize 8 in different ways. She also wanted to illustrate for students, ways of recording their observations with words and number sentences. After meeting with her cooperating teacher, she also realized that this task would allow them to see the extent to which students were subitizing. She planned to flash the image for 3 seconds, then again for another 3 sections and then discuss what students saw. After discussing the initial arrangement of dots (Formation 1), she rearranged the dots to see what else students may notice (Formation 2).



Formation 1



Formation 2

Some observations students made about Formation 1 were:

- *I noticed 5 on the top and 3 on the bottom.*
- *I counted really fast by 1’s.*
- *I knew it was a 10 frame and there were 2 spots left. That’s 8.*
- *I knew a whole row is 5 then 3 more is 8.*

Observations for Formation 2 were:

- *I noticed 4 on one side and 4 on the other. $4+4=8$.*

- *I saw that you didn't take any off.*
- *I know $2+2+2+2=8$.*
- *It's still the same thing.*

When debriefing the lesson, both the candidate and cooperating teacher were impressed with the children's thinking and participation. They decided that the Number Talk was a simple yet effective approach to gather children's ideas even though they did not hear ideas from every child. It had helped that the candidate asked the children to show a thumbs-up signal to let her know they understood or used the same strategy as the student sharing. The cooperating teacher pointed out that the children were subitizing to 10 and connecting their counting strategies to what they know about the operations and ten-ness. The candidate and cooperating teacher discussed ways that the candidate could more clearly record the children's ideas (i.e., allowing more space to add connections, changing pen color with different ideas) and how together they could continue to develop the children's skills with listening and making sense of others' ideas.

The purposes of any assessment should determine the content, the methods of collecting evidence, and the nature of the possible consequences for individual students, teachers, schools, or programs. Assessment that supports early childhood learning includes both formal and informal assessments and draws upon a range of sources of evidence. It should enhance teachers' powers of observation and understanding of children's mathematical thinking allowing them to establish individual learning goals. Strategies should include: individual assessment, observations, documentation of children's talk, interviews, samples of student work, and performance assessments with the intent to illuminate children's thinking, strengths, and needs.

Collaboration with Families Enhances Children's Mathematical Development

Well-prepared beginning teachers at the early childhood level collaborate with families in a mutually respectful, reciprocal way, to enhance and connect children's in-school and out-of-school mathematical development. [Elaboration of C.2.5]

Early childhood teachers have the ability to positively influence the relationship between families and schools and the attitudes of their children toward school. They do this by connecting and communicating with families in culturally sensitive ways. They develop home-school communication that allows two-way sharing of information, concerns, and feelings. They get to know families, develop trusting relationships and collaborate on behalf of their children. They use multiple methods to communicate the mathematics their children are learning and invite families to share the ways they were taught mathematics when they were children. By inviting parents to share their ways of doing mathematics, teachers learn to communicate how their learning is similar and different from the ways their children are learning mathematics in school. By connecting with parents, well-prepared beginning teachers engage in the mutual sharing of resources and ideas to support the mathematical development of young learners.

Well-prepared beginning teachers demonstrate interest in learning how their students' and families use mathematics at home and in their communities. For example, teachers can ask families to teach them the first ten number names in their home language (e.g., uno, dos, tres.) and use the names as the class counts small quantities. They can also invite families to collect home artifacts for the classroom to explore mathematical concepts. For example, empty boxes of commonly used household products can be used when discussing geometric shapes. These activities incorporate the child's culture in the

classroom and provide cultural and learning experiences for all students. Well-prepared beginning teachers forge partnerships with families and support them with knowing how to create and sustain learning opportunities at home. They may also support joint and separate sessions for parents and children at the school, including “bridging” activities for parents to develop their child’s numeracy at home (Doig, McCrae, & Rowe, 2003).

Standard C.3. Knowledge of Students as Learners of Mathematics

Effective teachers understand how students’ mathematical ideas develop and how to apply such understandings to every aspect of teaching. Such understanding is particularly important at the early childhood level, as children often interpret mathematical situations, even those that seems obvious to adults, quite differently. Therefore, we have a critical elaboration of this standard for early childhood mathematics education.

Seeing Mathematics through Children’s Eyes

Well-prepared beginning teachers at the early childhood level are conversant in the developmental progressions that are the core component of learning trajectories, thus seeing mathematical situations through children’s eyes. [Elaboration of C.3.1]

Well-prepared beginning teachers of mathematics have foundational understandings of students’ mathematical knowledge, skills, and dispositions. They also know how these understandings can contribute to effective teaching and are committed to expand and deepen their knowledge of students’ as learners of mathematics.

Indicators include:

- C.3.1. Students’ thinking about mathematics content
- C.3.2. Students’ engagement in mathematical practices
- C.3.3. Students’ mathematical dispositions

The younger the child the more important it is that teachers use children’s thinking and learning as a starting point. Further, the younger the child, the more difficult it can be to “to de-center” and see mathematical situations through children’s eyes. Fortunately, we know in great detail how children think and learn mathematics in the early years, and it is important that prospective teachers become conversant in these *developmental progressions* and utilize them as they plan for and interact with children in preschool and primary settings.

These developmental progressions are paths most children follow in learning a mathematics topic. These paths are children’s natural ways of learning. For example, consider that children first learn to crawl, then walk, then run, skip, and jump with increasing speed and dexterity. These are the levels in that developmental progression of movement. Children similarly follow natural developmental progressions in learning the concepts and skills within a certain domain or topic of mathematics. When teachers understand these developmental progressions, and select, sequence, and modify activities based on them, they create mathematics learning environments that are particularly developmentally appropriate and effective. These developmental progressions, then, are the core of a learning trajectory (which includes, as described previously, the mathematical goal, or content, and instructional activities and strategies corresponding to each level of the developmental progression).

Developmental progressions begin when life begins. Young children have certain mathematical-like competencies in number, spatial sense, and patterns from birth. As they develop and learn, they progress through identifiable *levels of thinking*—periods of time of qualitatively distinct patterns of thinking about mathematics. As an example, children develop increasingly sophisticated counting strategies to solve increasingly difficult types of arithmetic problems. For example, even very young

children, shown 1 chip on a plate covered, then shown another chip placed under the cover, can “make your plate look just like mine”—an early “visual addition.” Later, these children use a counting-all procedure. Given a situation, such as combining 6 red apples and 2 green apples, children count out objects to form a set of 6 items, then count out 2 more items, and finally count all those items and say “8.” After children develop such methods, they eventually fade them, in favor of other methods. On their own, children as young as 4 or 5 years may start “counting on,” solving the previous problem by counting, “Siiiiix... seven, eight. Eight!” The elongated pronunciation of the first addend substitutes for counting the initial set one-by-one. That approach is as if they counted a set of 6 items.

Thus, counting skills—especially sophisticated counting skills—play an important role in developing competence with computation. Counting-on when increasing collections and the corresponding counting-back-from when decreasing collections are critical numerical strategies for children. However, they are only beginning strategies. In the case where the amount of increase is unknown, children use counting-up to find the unknown amount. If five items are increased so that there are now nine items, children may find the amount of increase by counting and keeping track of the number of counts, as in “Fiiiive; 6, 7, 8, 9. Four (as in four counts, the amount of increase)!” And if nine items are decreased so that five remain, children may count back from nine down to five to find the unknown decrease, as follows: “Niiiiine; 8, 7, 6, 5, 4. Four!!” However, counting backwards, especially more than two or three counts, is difficult for most children unless they have consistent instruction. Instead, children might learn counting-up to the total to solve a subtraction situation. For example, “I took away 5 from 9, so 6, 7, 8, 9 (raising a finger with each count)— that’s 4 more left in the 9.” Learning this way, including the complementary use of number composition and decomposition strategies (e.g., “break apart to make a 10”) is more developmentally appropriate and more effective at achieving fluency than jumping immediately to verbal memorization (Baroody, 1999; Henry & Brown, 2008).

Standard C.4. Social Contexts of Mathematics Teaching and Learning

Effective teachers are attuned to the specific strengths and backgrounds of each of their students. They build on students’ current mathematical ideas and on students’ ways of knowing and learning, including attention to students’ culture, race/ethnicity, language, gender, socioeconomic status, cognitive and physical abilities, and personal interests. They also attend to developing positive mathematical identities and agency among their students. Such understanding is particularly important at the early childhood level, as children take their first steps from their home lives to the world of formal education. Therefore, we have a critical elaboration of this standard focused on the knowledge and contexts specific to the early childhood level.

Realizing that the social, historical, and institutional contexts of mathematics impact teaching and learning, well-prepared beginning teachers are knowledgeable about and committed to their critical role as advocates for every mathematics student.

Indicators include:

- C.4.1. Access and advancement
- C.4.2. Mathematical identities
- C.4.3. Students’ mathematical strengths
- C.4.4. Power and privilege in the history of mathematics education
- C.4.5. Ethical practice for advocacy

Creating Positive Early Childhood Learning Environments

Well-prepared beginning teachers at the early childhood level create mathematical learning environments, characterized by exploration, reasoning, and problem solving, that draws upon children’s mathematical, cultural, and linguistic strengths thereby developing conceptual understanding and positive mathematical identities. [Elaboration of C.4.2 & C.4.3]

Classroom learning environments provide a context to shape the way students experience and learn mathematics. From the visual displays on the classroom walls to the arrangement of desks and the accessibility of instructional materials, the learning environment communicates how a teacher views mathematics and what students are expected to learn and do. Well-prepared beginning teachers understand that learning environments impact young learners developing mathematical identities and can either support or hinder students' ability to learn mathematics. . These teachers create learning opportunities that invite all students to participate by using classroom routines as opportunities to explore mathematics. For example, as kindergartners are lined up for lunch, a teacher might facilitate an exploration of the various meaning of counting such as ordinal numbers (e.g., first, second, third in line) Similarly, a teacher may further develop first graders' understanding of measurement ideas by asking them to compare the heights of students in line and order them from tallest to shortest. Second graders may be asked to examine patterns and concepts of odd and even as they consider if every student has a partner when they form two lines. Whether students are counting out snacks to place in baggies or dividing classroom supplies for a mathematical task, well-prepared beginning teachers learn to attend to these events to deepen young children's learning of mathematical concepts.

Well-prepared beginning teachers understand the role of manipulative materials in helping young learners represent mathematical concepts and communicate their thinking. They select and use a variety of manipulatives materials and encourage students to use these tools s to explore mathematical ideas without always telling them when and how to use the materials. Materials are easily accessible for young learners to use as they explore mathematical concepts, solve problems, and communicate their mathematical thinking for themselves and others. Well-prepared beginning teachers also know that the type of manipulative materials and the names they give to them support students' later learning of mathematics. They use manipulatives to help young children make connections among concrete counters, number names, and symbols as children transition from play-based, real world knowledge of quantities to formal base ten number systems (Morin & Samelson, 2011).

Well-prepared beginning teachers establish classroom norms that value the various ways students explore and reason about mathematical situations and support young learners' development of mathematical explanations (Yackel and Cobb, 1996). They know that language and communication patterns are a function of a person's culture and understand students enter classrooms speaking the language and language vernaculars used by their families and friends outside of school. These teachers treat students' language as a resource, not a deficit and support all children, regardless of their English proficiency, to participate in class discussions (Moschkovich, 2010). Well-prepared beginning teachers see their role as helping young learners connect their out-of-school communication practices with the academic and mathematical language they are expected to use in schools. Well-prepared beginning teachers engage their students in classroom discussions that encourage students to use their existing communication patterns as they teach them the language of mathematics including vocabulary, symbols, and materials.

Vignette: Solving $21 + 32$

A class of first graders is finding the total number of stickers in two different sized packages of stickers. One package has 21 stickers and the other package has 32 stickers. The students can use connecting cubes, base-ten blocks, or paper and pencil to solve the problems. After the students solved the problem the teacher, Mr. Walker, engages the class in a discussion.

Teacher:	Who would like to share how they solved the problem? Rebecca?
Rebecca:	I used the sticks and cubes [Points to base-10 blocks]. I got two sticks and one cube and three sticks and two cubes and put them together. Then I got fifty-three.
Teacher:	How did you know to take out two tens and one unit to equal the number 21?
Rebecca:	Cause the stick is 10.
Teacher:	The stick is ten what?
Rebecca:	Ten tinies. So one stick is ten and two sticks are twenty and I needed one more to equals 21.
Teacher:	Will you count the tens and ones for us?
Rebecca:	[Touches each block as she counts] Ten, twenty, thirty, forty, fifty, fifty-one, fifty-two, fifty-three.
Teacher:	Thank you I see what you did now. Did anyone solve the problem a different way? Jamaal?
Jamaal:	I did it in my head. I plussed twenty and thirty and got fiddy. Then I plussed the one and the two and got 3. So my answer is fiddy-three.
Teacher:	[Writes 53 on the board]. So let me see if I understand what you did. First you added twenty and thirty and got the sum of 50. Then you added one and two and got the sum of 3. Then what did you do?
Jamaal:	I plussed, I mean added, <i>fiddy</i> and three and got fiddy-three.
Teacher:	I see, thank you for sharing.

In this exchange, Mr. Walker builds on Rebecca and Jamaal's everyday language and provides them with the mathematical terms to use as they communicate. Rather than correcting students as they speak, he re-voices their statements using mathematical terms. More importantly, in the case of Jamaal, Mr. Walker realized that he said "fiddy" instead of "fifty" but does not correct his language because he understood what Jamaal meant and does not want him feel uncomfortable sharing his thinking. Instead Mr. Walker used the correct mathematical pronunciation and continued the class discussion. Well-prepared beginning teachers build on children's mathematical, cultural and linguistic strengths to promote the positive and mathematical dispositions of every child.

Advocating for Every Young Child's Access to High-quality Mathematics Learning Experiences

Well-prepared beginning teachers at the early childhood level advocate for every child to have access to high-quality mathematical learning experiences in preschool and primary settings. [Elaboration of C.4.1, C.4.4, & C.4.5]

While beginning teachers may feel as though they have little effect on the policies that determine whether or not every child has access to attend a high-quality program, well-prepared beginners can positively influence the mathematical experiences of the children and community they serve. As described in other sections of this chapter, the beginning teachers' dispositions toward mathematics [Elaboration of C.1.3] and the learning environment they establish [Elaboration of C.4.2 & C.4.3] have a significant influence on the ways that children come to see themselves as mathematicians. Well-prepared beginners find ways to highlight each child as capable mathematics learners. They also seek ways to engage families as a means to improve mathematical learning both inside and outside the classroom.

Well-prepared beginning teachers can advocate for rigorous mathematics learning experiences so that all children may realize their full potential. They can do this by supporting other adults in seeing the

unique ways a child is building on their cultural and linguistic strengths to develop conceptual understanding of mathematics. They also help other adults understand that even if a child's thinking is incorrect or limited that there is much that is "right" about that thinking. The more skilled that well-prepared beginners are with understanding children's mathematical thinking and where that thinking fits along a developmental trajectory, the better they can design rich learning experiences and ask purposeful questions to support children's movement along that trajectory [Elaboration of C.3.1].

Part 2: Elaborations of the Characteristics and Qualities Needed by Effective Programs Preparing Early Childhood Teachers

This section provides additional detail of what preservice programs need to do to effectively prepare their students to teach early childhood mathematics. Although young children are ready and eager to learn (NRC, 2001), many early childhood teachers are not eager and prepared to engage children in rich experiences in domains other than literacy (Brenneman, Stevenson-Boyd, & Frede, 2009; IOM & NRC, 2015; NRC, 2007). Teachers of young children historically have not been prepared to teach domain-specific knowledge to young children (Isenberg, 2000). Programs should ensure that they follow all the recommendations in Chapter 3 to provide guidance in enacting research-based policies and practices for the preparation of all early childhood teachers of mathematics. Here we provide brief elaboration of only one standard, relating to clinical settings. The other standards are listed for the sake of completeness.

Standard P.1. Establish Partnerships

Effective programs for preparing mathematics teachers have significant input and participation from all appropriate stakeholders.

Indicators include:

- P.1.1. Engage all partners productively
- P.1.2. Provide institutional support

Standard P.2. Opportunities to Learn Mathematics

An effective mathematics teacher preparation program provides beginning teachers with opportunities to learn mathematics that are purposefully focused on essential big ideas across content and processes that foster a coherent understanding of mathematics for teaching.

Indicators include:

- P.2.1. Attend to mathematics content for teaching
- P.2.2. Build mathematical practices and processes
- P.2.3. Provide sustained experiences

Standard P.3. Provide Opportunities to Learn to Teach Mathematics

An effective mathematics teacher preparation program provides candidates with multiple opportunities to learn how to teach through mathematics-specific methods courses that integrate mathematics, knowledge of students as learners, the social contexts of mathematics teaching and learning, and practices for teaching mathematics.

Indicators include:

- P.3.1. Address deep and meaningful mathematics content knowledge
- P.3.2. Provide candidates with strong foundations of knowledge about students as mathematics learners
- P.3.3. Address the social contexts of teaching and learning
- P.3.4. Provide practice-based experiences
- P.3.5. Provide effective mathematics methods instructors

Standard P.4. Opportunities to Learn in Clinical Settings

The preparation program of beginning early childhood teachers of mathematics should include clinical experiences in both preschool and primary settings that are exemplar sites in equitable early mathematics learning. These sites illustrate responsive interactions with children individually, in small groups, and as a class.

Providing Diverse Clinical Settings: Preschool and Primary

Programs preparing teachers of young children include opportunities to learn in both preschool and primary settings. [Elaboration of P.4]

Experiences in both preschool and primary settings should include “field observations, field work, practica, student teaching and other ‘clinical’ practice experiences such as home visiting” (NAEYC 2010 Initial and Advanced Standards). These experiences provide opportunities for the systematic inquiry into classroom practice under the supervision of licensed professionals with the intention of preparing teachers of young children who develop nurturing, responsive relationships with children and families. Additional recommendations can be found in Standard 3.4 Opportunities to Learn in Clinical Settings.

Programs benefit from developing partner schools that host groups of teacher candidates pairing them with cooperating teachers who are willing to open their classroom practice in ways that benefit learning for the young learners, the teacher candidates, and the cooperating teacher themselves. Such a setting provides a space for close examination of instructional practice as highlighted in the vignette. Teacher candidate/cooperating teacher teams collaboratively plan, teach, and debrief lessons based on student data (Rigelman & Ruben, 2012). When purposefully designed, the clinical experiences can be explicitly linked to the mathematics methods coursework which narrows the theory-practice gap lamented by many teacher educators (Darling-Hammond & Bransford, 2007). These strong partnerships provide a space for preservice and inservice teacher learning in support of improved student mathematics learning.

The clinical experiences of effective teacher preparation programs are guided by a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for teacher candidates to teach and to learn from their own teaching and the teaching of others.

Indicators include:

- P.4.1. Collaboratively develop and enact clinical experiences
- P.4.2. Sequence school-based experiences
- P.4.3. Experience teaching with diverse learners
- P.4.4. Recruit and support qualified mentor teachers and teacher preparation supervisors

Standard P.5. Recruit and Retain Teacher Candidates

An effective mathematics teacher preparation program attracts, nurtures, and graduates high quality teachers of mathematics who are representative of diverse communities.

Indicators include:

- P.5.1. Recruit strong candidates
- P.5.2. Address diverse community needs
- P.5.3. Provide experiences and support structures
- P.5.4. Assess recruitment and retention data

Closing Remarks

Effective teacher preparation programs at the early childhood level must develop candidates' abilities to use high-leverage, effective mathematics teaching practices (NCTM, 2014), which requires a deep understanding of the mathematics they expected to teach. The teaching they are expected to enact on a daily basis with young learners often stands in sharp contrast to what many prospective teachers experienced themselves as learners of mathematics (Isenberg, 2000). They often describe their own experiences as being teacher-centered instruction emphasizing memorization of facts and procedures with little to no emphasis on understanding, problem solving, reasoning, and application. In addition, many early childhood teachers report high levels of mathematics anxiety and avoidance. Effective programs consciously break the insidious cycle that currently exists in which early childhood teachers pass their own anxieties and superficial knowledge of mathematics on to their students. Instead, effective programs prepare their prospective teachers to engage in "ambitious teaching." That is, "teaching that aims to teach all kinds of students to not only know academic subjects, but also to be able to use what they know in working on authentic problems in academic domains" (Lampert, Boerst, & Graziani, 2011, p. 1). In addition, effective programs instill in their prospective teachers a positive and productive disposition toward mathematics teaching and learning, which they in turn pass on to their future students.

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CHAPTER 5. ELABORATIONS OF THE STANDARDS FOR THE PREPARATION OF TEACHERS OF MATHEMATICS FOR GRADES THREE THROUGH FIVE

Teachers who teach mathematics in grades three through five must not only have strong teaching skills but also have strong content knowledge, strong knowledge of mathematics-specific pedagogy, and much more, including cultural knowledge about their individual students, school policies, and how to collaborate with other teachers. Only with this knowledge will mathematics teachers be able to meaningfully support the learning of all students.

Building on the standards presented in Chapters 2 and 3, this chapter puts forth particular standards for the knowledge, skills, dispositions, and actions of well-prepared beginning grades three through five teachers of mathematics, as well as what preservice programs need to do to ensure the effective preparation of candidates to develop those necessary knowledge, skills, dispositions, and actions. Additionally, the chapter provides commentary and examples about those standards relevant to grades three through five. The chapter concludes with standards regarding how programs may achieve these standards.

Table 5.1 lists the elaborations of the standards presented in Chapters 2 and 3 as they relate to preparing mathematics teachers for grades three through five.

Table 5.1. Elaborations of Candidate Standards for Teachers of Mathematics for Grades Three Through Five

Part 1: Candidate Knowledge, Skills, and Dispositions

Understand the Mathematics Concepts and the Connection of Mathematical Practices to those Concepts.

Well-prepared beginning grades three through five teachers of mathematics understand foundational mathematics concepts that they will teach and connect those concepts to mathematical practices as well as to pre-K – 2 mathematics and the mathematics of the middle-school curriculum. [Elaboration of C.1.1 and C.1.2]

Develop Pedagogical Knowledge and Teaching Practices

Well-prepared beginning grades three through five teachers of mathematics develop pedagogical knowledge and practices to cultivate students' mathematical proficiency including such components as conceptual understanding, procedural fluency, problem-solving ability and their facility with the mathematical processes essential for learning. *[Elaboration of C.2.2 and C.2.3]*

Use Tools to Build Student Understanding

Well-prepared beginning grades three through five teachers of mathematics effectively use *technology* tools, physical models, and mathematical representations to build student understanding of the topics at these grade levels. *[Elaboration of C.1.6 and C.2.3]*

Use Assessment to Promote Learning and Improve Instruction

Well-prepared beginning grades three through five teachers of mathematics learn how to use both formal and informal assessment tools and strategies to gather evidence of students' mathematical thinking in ways appropriate for young learners, such as the use of observation and interview protocols, questioning sequences, paper-and-pencil tasks, computer-based tasks, and digital records (audio and video). These assessments include progress monitoring of students through diagnostic interviews and other means. [Elaboration of C.3.1]

Support Students' Sense-Making

Well-prepared beginning grades three through five teachers of mathematics nurture children's proficiency with, and sense-making of, mathematical ideas, processes, and practices. They draw on knowledge gathered through assessments of children's mathematical ideas and dispositions, as well as knowledge from research of how mathematical ideas, as well as knowledge from research of how mathematical ideas develop over time. [Elaboration of C.3.1 and C.3.2]

Become Ethical Advocates for Children

Well-prepared beginning grades three through five teachers of mathematics understand their role as ethical advocates for elementary children to have access to and advance in mathematics that cultivates a positive mathematics identity, connects to children's mathematical thinking and lived experiences, build partnerships with families and communities, and work to eliminate institutional and curricular barriers to learning. [Elaboration of C.4.1]

Part 2: Program Characteristics and Qualities

Mathematical Content Preparation of Grades 3 – 5 Teachers of Mathematics

Programs that support the development of well-prepared beginning mathematics teachers of grades three through five should include coursework and other experiences focused on key mathematical ideas and skills that are pivotal in those grades. [Elaboration of P.2.1]

Mathematics Pedagogy Preparation of Grades 3 – 5 Teachers of Mathematics

Programs that support the development of well-prepared beginning mathematics teachers of grades three through five should include coursework that supports the development of practices, techniques, and habits of mind or dispositions that support the integrity of the mathematics that is taught and the mathematical development of each student. [Elaboration of P.3.2]

Mathematics Methods Coursework for Grades 3 – 5 Teachers of Mathematics

Programs that support the development of well-prepared beginning mathematics teachers of grades three through five should include at least one mathematics methods course, or the equivalent of three semester units, focusing particularly on mathematics teaching and learning in grades three through five. [Elaboration of P.3.1, P.3.2, P.3.3, and P.3.4]

Clinical Experiences for Grades 3 – 5 Teachers of Mathematics

Programs that support the development of well-prepared beginning mathematics teachers of grades three through five develop and systematically use a collection of clinical settings that support beginning teachers work with diverse learners, curricula, and within different institutional contexts. [Elaboration of P.4.2 and P.4.3]

Part 1: Elaborations of the Knowledge, Skills and Dispositions Needed by Well-prepared Beginning Mathematics Teachers for Grades Three through Five

This section provides additional detail, commentary, and examples of the knowledge, skills, and dispositions well-prepared beginning teachers who teach mathematics in grades three through five should have, organized by the general standards described in Chapter 2. Standards specific to teaching mathematics in grades three through five are defined and described, along with additional commentary and examples about the general standards in Chapter 2.

Standard C.1. Knowledge of Mathematics for Teaching

Effective teachers have a deep understanding of the mathematics they are expected to teach and exhibit positive dispositions toward both mathematics teaching and learning. Such understanding and dispositions are particularly critical for early childhood teachers, as they develop the foundation of mathematical understanding, beliefs, and attitudes among young learners that start students on their mathematical journey. Therefore, we have a critical elaboration of this standard for the preparation of early childhood teachers of mathematics.

Understand the Mathematics Concepts and the Connection of Mathematical Practices to those Concepts.

Well-prepared beginning grades three through five teachers of mathematics understand foundational mathematics concepts that they will teach and connect those concepts to mathematical practices as well as to pre-K – 2 mathematics and the mathematics of the middle-school curriculum. [Elaboration of C.1.1 and C.1.2]

Well-prepared beginning teachers of mathematics possess appropriate mathematical knowledge of and skill in mathematics needed for teaching. They engage in appropriate mathematical practices and support their students in doing the same. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students' mathematical productions.

Indicators include:

- C.1.1. Core content knowledge
- C.1.2. Mathematical practices and processes
- C.1.3. Mathematical dispositions
- C.1.4. Analyzing the mathematical content of curriculum
- C.1.5. Mathematical knowledge to analyze mathematical thinking
- C.1.6. Using mathematical tools and technology

Prospective grades three through five elementary teachers often enter their preservice teacher education courses with fragile mathematics identities. Many of them voice concerns about teaching mathematics because of their own, often negative, mathematics instructional experiences in PK-12 education. Or, they may feel a sense of relief because they believe that learning elementary mathematics means knowing how to compute with the basic operations. They may think that because they know how to multiply, they can teach multiplication. Like many people, prospective teachers are unaware of the depth and complexity of elementary mathematics concepts. Even though the standard algorithms often continue to be emphasized in elementary schools, prospective teachers may not know that the familiar algorithms that they learned in school and often used without understanding can and often are taught by linking these procedures to important mathematical structures and properties. Thus developing the foundations of a robust mathematical knowledge base is essential for learning how to effectively teach elementary mathematics.

Even well-prepared beginning teachers do not learn all of the mathematics content they need to teach all elementary grade levels in their preservice teacher education program. However, there are some key areas that they should study, and they should have an opportunity to study at least some of them in

depth. Key areas at grades three through five include base-ten numbers, multiplicative structures, fractions and decimals, algebraic thinking, measurement and geometry concepts.

In brief summaries below, we describe the significant concepts that well-prepared beginning teachers of mathematics must know in order to be able to support learners in grades three through five and how those concepts connect with mathematical practices. The summaries include specific, but not exhaustive, examples of what it means to understand and be ready to teach this content. These ideas connect to and reflect the MET II content expectations (Conference Board of Mathematical Sciences (CBMS), 2012) and therefore, we include the related MET II content expectations in each section in addition to research in mathematics teacher education. This section was influenced by representations of core understandings that students in grades three through five develop found in documents such as NCTM's *Developing Essential Understandings* series and *Curriculum Focal Points*, and the Common Core State Standards. We use these resources to project mathematical knowledge and knowledge of mathematical practices needed by teachers of children in these grades, not to equate the deeper and more flexible knowledge that teachers need to support children who are beginning to develop in these key areas.

Multiplicative Structures. Well-prepared beginning teachers of mathematics in grades three through five are multiplicative thinkers, with deep understandings of the following concepts and topics:

- Multiplication and division have meaning, including several different interpretations, and are more than just memorization of basic facts and procedures. Sometimes teachers (and students) believe that multiplication is merely repeated addition or equal groups and division is only making groups, but well-prepared beginning teachers must be familiar beyond those partial understandings with the multiple meanings of and real-world contexts for multiplication and division. Well-prepared beginning teachers can represent these operations in many ways and can make connections between representations and problem-types.
- Properties, such as the commutative, associative, and distributive properties, support justification, flexibility, and fluency, and help students make sense of multi-digit computation.
- Computation involving multiplication and division should include mental computation, estimation strategies, invented algorithms, and standard algorithms (Otto, Caldwell, Lubinski, and Hancock, 2011).

The concepts of multiplication and division must be deeply understood by well-prepared beginning grade three through five teachers, and this understanding must include multiple representations of the concepts as well as how to sequence and teach this content to students. This includes recognition of the relationship between the content that precedes multiplication and division (e.g., addition and subtraction and place value) and the content that follows multiplication and division (e.g., ratios and proportions). The movement from additive thinking, particularly counting by ones, to multiplicative thinking or seeing an equal group as a unit is challenging for students and the well-prepared beginning teacher should be able to predict and respond to that potential barrier.

Well-prepared beginning grade three through five teachers know that the way to approach this content is through engaging students in reasoning about situations involving multiplication and division. This presents opportunities to connect the action described in the problem with the operations, which is also essentially the definition of the mathematical practice of making sense of problem situations. Well-prepared beginners know how to select and sequence problems that introduce students to a range of interpretations of these concepts and the need to develop strategies for solving these problems through multiple approaches. These approaches include skip counting, equal groups, area and array models, multiplicative comparisons, ratio tables, scaling, and partitive and measurement division situations

(Otto, Caldwell, Lubinski, and Hancock, 2011). Additionally, well-prepared beginning teachers are able to select rich tasks that lead students to the use of multiple approaches, connect to relevant contexts, and connect to other content within mathematics.

Figure 5.1 lists items from the MET II report (CBMS, 2012) related to multiplicative structures in grades three through five.

Figure 5.1. Connections to the CBMS (2012) Report on *The Mathematical Education of Teachers II* Related to Multiplicative Structures in Grades Three Through Five

MET II (CBMS, 2012) describes the following knowledge related to multiplicative structures:

- “The different types of problems solved by multiplication and division, and meanings of the operations illustrated by these problem types.
- Teaching–learning paths for single-digit multiplication and associated division, including the use of properties of operations (i.e., the field axioms)” (p. 26)
- “Recognizing that addition, subtraction, multiplication, and division problem types and associated meanings for the operations (e.g., CCSS, pp. 88–89) extend from whole numbers to fractions” (p. 28).

Fractions and Decimals. Well-prepared beginning teachers of mathematics in grades three through five have a strong understanding of fractions and decimals, including the following concepts and topics:

- Fractions have multiple interpretations, including part-whole relationships, measures, quotients, ratios, and operators.
- The unit is a foundational concept, “fundamental to the interpretation of rational numbers” (Otto et al., 2011, p. 8).
- Equivalence is a key concept. Fractions can be expressed in an infinite number of equivalent fractions and in decimal form.
- Understanding the magnitude of fractions and decimals allows students to compare and order these numbers and allows for computational estimation.
- Computation with fractions and decimals builds on understanding of whole number operations, but some interpretations and contexts make more sense. Estimation and mental math continue to be important.

Grades three through five students experience shifts in their mathematical thinking that include:

- Shifting from discrete, countable quantities to continuous quantities.
- Shifting from one model to a variety of representations, flexibly thinking about the unit.
- Shifting from whole-number-based comparisons to equivalence-based comparisons
- Shifting from rules to making sense about operations on fractions (Otto et al., 2011).

As students experience the natural disequilibrium that occurs when making these shifts, well-prepared beginning teachers are ready to support sense making and understanding. This is a context in which an explicit emphasis on mathematical practices can play a key role. Well-prepared beginners encourage students to communicate their reasoning, critique the reasoning of others, and develop arguments through discourse and mathematical writing. Particularly at the third grade, students will learn that an argument is a carefully crafted sequence of statements and reasoning strategies presented with an objective to convincing others that a claim is true or false, rather than merely a written listing of

procedural steps taken in a process or algorithm to arrive at an answer. Students in grades three through five are formally introduced to mathematical argument, including a claim, justification of the claim with evidence, and warrants, that connect the reasoning and evidence to the claim (Casa, 2016). This introduction helps set the foundation for mathematical arguments in later grades including inductive and deductive proof and the analysis, representing, reasoning, revising and reporting demands of mathematical modeling at the high school level.

Figure 5.2 lists items from the MET II report (CBMS, 2012) related to fractions and decimals in grades three through five.

Figure 5.2. Connections to the CBMS (2012) Report on *The Mathematical Education of Teachers II* Related to Fractions and Decimals in Grades Three Through Five

MET II describes the following knowledge related to fractions and decimals:

- “Understanding fractions as numbers that can be represented with lengths and on number lines.

Using the CCSS development of fractions to define fractions $\frac{a}{b}$ as a parts, each of size $\frac{1}{b}$.

Attending closely to the whole (referent unit) while solving problems and explaining solutions.

- Recognizing that addition, subtraction, multiplication, and division problem types and associated meanings for the operations (e.g., CCSS, pp. 88–89) extend from whole numbers to fractions.
- Explaining the rationale behind equivalent fractions and procedures for adding, subtracting, multiplying, and dividing fractions. (This includes connections to grades 6–8 mathematics.)
- Understanding the connection between fractions and division, $\frac{a}{b} = a \div b$, and how fractions, ratios, and rates are connected via unit rates. (This includes connections to grades 6–8 mathematics. See the Ratio and Proportion Progression for a discussion of unit rate.)” (CBMS, 2012, p. 28)
- “Extending the base-ten system to decimals and viewing decimals as address systems on number lines. Explaining the rationales for decimal computation methods. (This includes connections to grades 6–8 mathematics.)” (CBMS, 2012, p. 27)

Geometry and Measurement. Well-prepared beginning teachers of mathematics in grades three through five have a strong understanding of geometry and measurement. They understand core measurement concepts, such as iteration, conservation, and origin, and provide a framework for connecting linear measurement with measures of area and volume (Clements & Sarama, 2014). They can make correspondences between direct measurement of properties of shapes and algebraic approaches to determine the same measures. They can harness ideas such as line symmetry and reflection to solve problems and as tools for thinking about fractions, area, and proportions. Lehrer and Slovin (2014) note the importance of:

- “Transforming objects and the space that they occupy in various ways while noting what does and does not change provides insight into and understanding of the objects and space” (p. 8).
- Measuring attributes is one way to analyze and describe geometric shapes.
- Classifying properties of objects helps investigate relationships between types of objects.

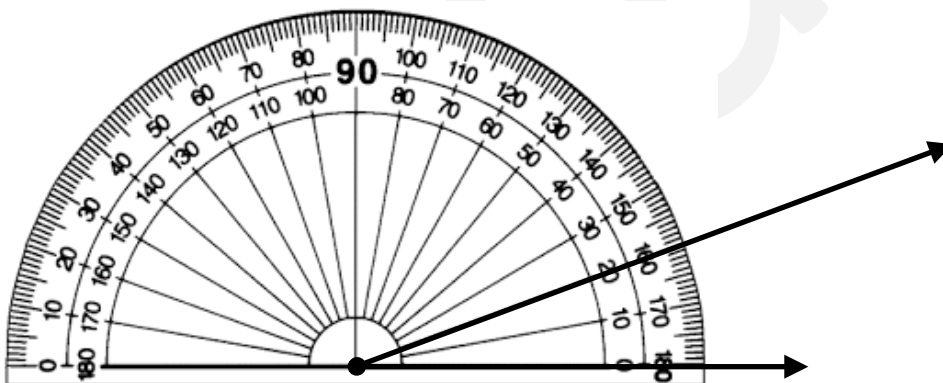
Well-started beginners teaching grades three through five can engage meaningfully in these mathematical topic areas through the mathematical practices. For example, these teachers are not only able to use measurement tools and construct figures, they are skilled at describing how to select appropriate tools. They also know how to highlight the decision making process to students explaining how tools are used to provide accurate information. For example, well prepared beginning teachers of grades three through five know how to use a protractor and also know how to talk about and show the use of this measuring tool in such a way that they could help students work through the array of challenges that often arise when using a protractor.

The vignette below showcases struggles that students sometimes have when using a protractor.

Vignette: Measuring Angles

Mr. Wu is watching his students use protractors to measure a variety of angles. On a seating chart, he records what he observes small groups of students doing and key points he hears them discussing. He also sketches what the students' protractors looks like in relation to the angle being measured. Some of the information he records includes:

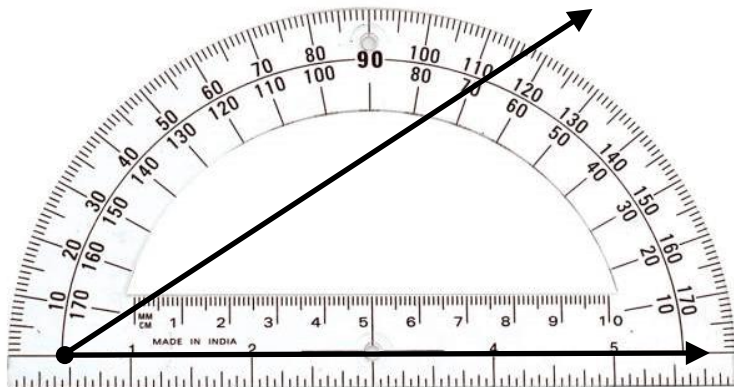
Group 1: "We are doing everything right, but now there are two numbers, 20 and 160 on the protractor. Which number do we use?" (see image)



Group 2: Hearing the comment, Chloe says to group 1: "Move your protractor and just look at the angle. Is it bigger or smaller than a square corner? If the angle is smaller, you pick the smaller number, because a square corner is 90 degrees."

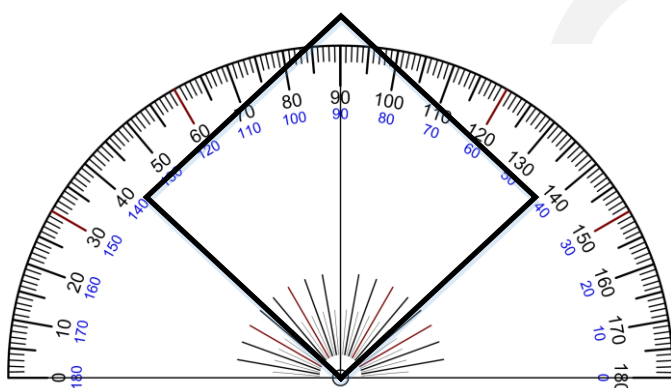
Group 3: Vince: "Where are you supposed to put the little circle?" J'La: "You put it where both of the arrows start."

Group 4: "When I put the protractor on the bottom arrow I get 5 inches. Is that how long the angle is?" (see image)



Group 5: Brittany: “This angle is too short to measure. The lines don’t reach the numbers on the protractor” Diego draws the line from the angle to the scale on the protractor and says: “See you just make the sides long enough to get to the numbers.”

Group 6: Frustrated... “I don’t get this, you can get any number. You have the little circle on the vertex and you can just move the protractor around like this to get any number...” (see image)



Mr. Wu anticipated many of the challenges his students encountered and wants them to collectively make sense of how to effectively use the protractor. Using his notes he lists the questions and big ideas on the front board. Then from his observations he tentatively plans who he will ask to share in the whole class discussion, including students who present challenges for the class to think about and also students he could strategically call on to contribute ways of thinking that could move the discussion forward. He is ready to help frame the by assisting students in using the document camera to show how the protractor and angle looked in different situations, to support the precise use of mathematical language, and to involve many students to build off of, question, and critique what is said.

Another example of well-prepared beginning teachers’ use of mathematical practices in this area is through the way in which they are able to attend to precision. Well-prepared beginning teachers of grades three through five know the different affordances of referring to a square as a rectangle, rhombus, parallelogram, or quadrilateral. They know and can generate contexts that require measurement of time to the nearest hour, minute, or tenth of a second. The well-prepared beginner of

grades three through five also knows that decisions about the precision needed will affect the selection and use of particular tools or representations. If measuring to the nearest inch will suffice, there is no real need to use a ruler partitioned to show 16th of an inch increments. If weighing a very light object in a science experiment is the goal, something more sensitive than a bathroom scale will be needed. The well-prepared beginner knows this, can make choices of tools and degrees of precision and justify their choices and the sensibility of their eventual measurements.

Figure 5.4 lists items from the MET II report (CBMS, 2012) related to geometry and measurement in grades three through five.

Figure 5.4. Connections to the CBMS (2012) Report on *The Mathematical Education of Teachers II* Related to Geometry and Measurement in Grades Three Through Five

MET II describes the following knowledge related to geometry and measurement:

- “Understanding geometric concepts of angle, parallel, and perpendicular, and using them in describing and defining shapes; describing and reasoning about spatial locations (including the coordinate plane).
- Classifying shapes into categories and reasoning to explain relationships among the categories.
- Reason about proportional relationships in scaling shapes up and down. (This is a connection to grades 6–8 geometry.)” (CBMS, 2012, p. 30).
- “The general principles of measurement, the process of iterations, and the central role of units: that measurement requires a choice of measureable attribute, that measurement is comparison with a unit and how the size of a unit affects measurements, and the iteration, additivity, and invariance used in determining measurements.
- How the number line connects measurement with number through length (see the Geometric Measurement Progression).
- Understanding what area and volume are and giving rationales for area and volume formulas that can be obtained by finitely many compositions and decompositions of unit squares or unit cubes, including formulas for the areas of rectangles, triangles, and parallelograms, and volumes of rectangular prisms. (This includes connections to grades 6–8 geometry, see the Geometric Measurement Progression.)
- Using data displays to ask and answer questions about data. Understanding measures used to summarize data, including the mean, median, interquartile range, and mean absolute deviation, and using these measures to compare data sets. (This includes connections to grades 6–8 statistics, see the Measurement Data Progression.)” (CBMS, 2012, p. 29).

Algebraic Thinking. Well-prepared beginning teachers of mathematics in grades three through five have a strong understanding of algebraic thinking, including the following concepts and topics (Blanton et al., 2011):

- The fundamental properties of arithmetic, such as the commutative, associative, and distributive properties, also hold for algebra.
- Equations represent an equivalent relationship.
- Variables can be used to describe mathematical ideas and may have different meanings.
- Quantitative reasoning helps make generalizations about relationships.

Well-prepared beginning teachers understand children's intuitive strategies and how those strategies build on and connect with the properties of operations and other algebraic concepts (Carpenter et al., 2003). Well-prepared beginning teachers fluently use mathematical symbols and conventions to express mathematical ideas. They can readily translate and contextualize symbolic representations of phenomena, as well as notice mathematical relations and patterns within real life and problem contexts that can be expressed more generally and/or abstractly. Importantly they are well positioned to help students engage in mathematical practices in which thinking quantitatively is linked with opportunities to reason abstractly. Representations such as drawings, schemas, and equations are vehicles that students can use to carry their thinking into more abstract realms. As they mature, grades three through five students are increasingly able to record their thinking, so beginning teachers make the recording of thinking and reasoning an integral part of instruction and assessments at these grade levels.

Well-prepared beginners make connections between algebraic representations and graphs and tables of the same situation, as well as discuss the relative advantages of the different representations. They use graphs and tables to give meaning to algebraic expressions and can draw students' attention to the elegance and power of an algebraic expression to model the mathematics of a situation. They know when and how an algebraic representation can be used to capture the logic of a numerical pattern and to show the mathematical structure of a situation.

Figure 5.5 lists items from the MET II report (CBMS, 2012) related to algebraic thinking in grades three through five.

Figure 5.5. Connections to the CBMS (2012) Report on *The Mathematical Education of Teachers II* Related to Algebraic Thinking in Grades Three Through Five

MET II describes the following knowledge related to algebraic thinking:

- “Recognizing the foundations of algebra in elementary mathematics, including understanding the equal sign as meaning ‘the same amount as’ rather than a ‘calculate the answer’ symbol” (CBMS, 2012, p. 26).

Standard C.2. Knowledge and Pedagogical Practices for Teaching Mathematics

Teaching mathematics is complex. It entails not only knowing the mathematics, but also knowing how to design and implement rich mathematics learning experiences that advance students' mathematical knowledge and proficiencies. Effective teachers are skilled in their use of high-leverage mathematics teaching practices and use those pedagogical practices to guide both their preparation and enactment of mathematics lessons. The development of these content-focused skills and abilities should form the core of work in the preparation of mathematics teachers for grades three through five. The following section elaborates on the knowledge and pedagogical practices specific to the teachers for grades three through five.

Develop Pedagogical Knowledge and Teaching Practices

Well-prepared beginning grades three through five teachers of mathematics develop pedagogical knowledge and practices to cultivate students' mathematical proficiency including such components as conceptual understanding, procedural fluency, problem-solving ability and their facility with the mathematical processes essential for learning. [Elaboration of C.2.2 and C.2.3]

Well-prepared beginning teachers of mathematics have a foundation of pedagogical content knowledge, effective and equitable mathematics teaching practices, and a positive and productive disposition toward teaching mathematics to support students' sense-making, understanding, and reasoning.

Indicators include:

- C.2.1. Promoting equitable teaching
- C.2.2. Planning for effective instruction
- C.2.3. Implementing effective instruction
- C.2.4. Analyzing teaching practice
- C.2.5. Enhancing teaching through collaboration with colleagues, families, and community members

Central to any efforts to deliver high quality instruction to all students is the development of pedagogical knowledge (Grossman, 1990; Shulman, 1986). While Shulman pointed to two categorical groupings of pedagogical knowledge: general pedagogical knowledge and pedagogical content knowledge (PCK), both Ball et al. (2008) and Sowder (2007) make more specific connections to how PCK plays out when considering the unique components and elements essential to teaching mathematics. The well prepared beginning teacher at grades three through five can pull apart the meaning of the standard algorithms which are largely taught at these grade levels including the particular models that link most effectively and the ways to let students generate the procedure themselves through multiple experiences where they record their steps. They help intermediate elementary students learn to select appropriate tools and use them to engage in mathematical practices such as seeing patterns and structure. Well-prepared beginning teachers also are fully cognizant of the common errors and naive conceptions that emerge from not only these procedures with whole numbers, fractions and decimals but the larger concepts in which they are embedded. Of course the deeper the mathematical background of the well-prepared beginning teacher, the greater their potential for showing pedagogical sophistication (Holm & Kajander, 2012). Holm and Kajander suggest that when dividing a whole number by a fraction the teacher should be able to pick the best problem to “see” the repeated subtraction approach, the strongest visual model, the most salient way of discussing the linkages back to prior knowledge about division of whole numbers and the debriefing of how the answer should be interpreted.

The representations, notation, strategies and language that are used in the classroom drive grades three through five students' understanding of procedures and concepts. Well-prepared beginning teachers align with conventions for proper notation, such as distinguishing between a multiplication symbol and

the variable x , and precise language, such as using the verb “regroup” rather than “borrow” so there is a consistent message to students across all three grades and a smooth path toward building on prior knowledge in meaningful ways that last. They recognize that rules or shortcuts, such as just add a zero at the end of a number when multiplying by ten, that are taught but only work for a short time (or even not very well at all) should be avoided in favor of identifying patterns that students note and pointing out constraints or boundaries of the usage of those “rules” as they emerge (Karp, Bush, & Dougherty, 2014). This is particularly important in grades three through five, when many rules about whole numbers fall apart with the introduction of fractions and decimals (the longer the number the larger the number for instance). This cognitive dissonance between what was taught before as true (multiplication makes larger) and the same idea that now no longer “works” (multiplication of fractions) puts students in this age group at risk of thinking mathematics is mystical or at worst ever changing and unknowable.

The vignette below describes a beginning teacher working with students who were struggling with solving word problems.

Vignette: Using a Keywords Strategy to Solve Word Problems

Ms. Morgan was working with a small group of third graders who were having trouble solving word problems. She asked the students to meet to discuss their strategy use.

Nela was asked how she decided to use addition to solve the problem, **“There are three baskets of apples on the table. Each basket contains six apples. How many apples are there in all?”**

Nela responded that she saw the words “in all” and that meant that the numbers listed in the problem should be added. She arrived at an answer of 9. Rory suggested that for the problem, **“Each student was given an equal share of stickers. If there are 25 stickers and 4 students, how many stickers will each student receive?”**

Rory said, “You use the word “each” and then you know to multiply - so they each get 100 stickers.”

At this point Ms. Morgan realized both students were describing the use of key word strategies learned in previous grades. The rules or shortcut that began in primary years were causing serious issues in later grades. As in this case, sometimes students are mistakenly encouraged to skim through a word problem and locate the key words as a strategy to quickly choose an operation to solve the problem and which number(s) from the problem are relevant.

Ms. Morgan had seen lists of key words in other classrooms that linked particular words with corresponding operations – such as “each = multiply” and so on. But as seen with Nela and Rory, these words frequently do not indicate the operation that corresponds with the problem.

Further, the technique is unusable when problems don’t have key words or students consider multi-step problems. Ms. Morgan decided to show the students three word problems with the same key word where the problems would be solved using different operations so that she could discuss the pitfalls and limitations of the key word approach.

Instead, well-prepared beginning teachers focus on sense-making and reasoning as they prepare

students to grasp the full meaning of a problem by reading the entire situation and trying to use structures, such as schema, properties of the operations, and representations to come to a reasoned solution.

Key Instructional Supports. Well-prepared beginning teachers support the learning of all students. This is particularly important in Multi Tiered Systems of Support (MTSS) such as Response to Intervention (RtI), as students are usually identified for formal special education services starting in the third grade. This process requires the careful assessment of students to pinpoint their strengths and gaps so that instruction and interventions can be targeted – whether for students with disabilities or for students who are identified as “gifted” with a high interest in or a talent for mathematics. With reference to English Language learners, the well-prepared beginning teacher is able to incorporate the appropriate linguistic practices and strategies needed including home language connections and relevant academic language and discourse practices to support emerging multilingual students as they move to more complex mathematics vocabulary. Instruction builds on relevant contexts and the need to build on students’ lived experiences in and out of the school setting. All of this knowledge about and emphasis on teaching individual learners, precludes the use of curriculum interventions via generic computer programs, basic worksheets, or Internet searches for “attractive” ideas.

Aligning Mathematical Concepts Across the Grades

While all well-prepared beginning teachers strive to align mathematical concepts across the grades, this is particularly crucial for teachers of grades three through five who bridge work in primary grades and later work in such courses as Algebra I. A pressing challenge is teaching in ways that support the development of mathematical ideas over time while resisting the practice of teaching only the mathematics that appears in the standards for one’s own grade level. For example, well-prepared beginning teachers of grade five invest in knowing middle school content so that they are positioned to support students’ readiness even when some of those ideas are not well represented in the fifth grade standards. The idea of continuity of development certainly applies to teaching across grades three through five. For example, the responsibility for the use of number lines is represented most strongly in third grade standards. Well prepared beginning teachers in grades four and five picks up on the development and use of the number line even though it is not specifically articulated in the standards for their grade levels. In sum, well-prepared beginners have a strategic understanding of the trajectory of the representations used and takes responsibility for meeting grade-level standards and reinforcing what came before (the meaning of the equal sign) and what is still to come in later grades such as when fifth graders see the more sophisticated use of vertical and horizontal number lines with locating points on a coordinate graph.

Use Tools to Build Student Understanding

Well-prepared beginning grades three through five teachers of mathematics effectively use technology tools, physical models, and mathematical representations to build student understanding of the topics at these grade levels. [Elaboration of C.1.6 and C.2.3]

Well-prepared beginning teachers know when to use different manipulatives and various technologies to support students in developing mathematical concepts and to create opportunities for collective consideration of mathematical ideas such as multiplication, fractions, area, volume and coordinate geometry. They judiciously select particular representations based on mathematical considerations, knowledge of their students, and other relevant factors. For example, developing deep understanding of

fractions students must flexibly use three representations: area, linear measurement and set models. The set model is the most complex of the three representations, so well-prepared beginning teachers understand that modeling with fractions would start with area models and linear measurement models that connect to the number line followed by the set model. Furthermore, they flexibly and resourcefully think about what representations are available in their current classroom, school, and wider community, and also advocate for resources that will enhance their ability to convey mathematical ideas for students to explore and articulate their ideas. This might include helping students' utilize calculators responsibly, giving them access to operations with large numbers and decimals that calculating by hand would be extraordinarily cumbersome. They also understand that meaning is not inherent in a tool or representation, but that it needs to be developed through a combination of exploration, carefully orchestrated experiences, and explicit dialog focused on meaning-making (Ball, 1992). As a result they support students' connections between these representations – drawing links between and among equations, situations, manipulatives, and graphs developed through technology and other tools.

Use Assessment to Promote Learning and Improve Instruction

Well-prepared beginning grades three through five teachers of mathematics learn how to use both formal and informal assessment tools and strategies to gather evidence of students' mathematical thinking in ways appropriate for young learners, such as the use of observation and interview protocols, questioning sequences, paper-and-pencil tasks, computer-based tasks, and digital records (audio and video). These assessments include progress monitoring of students through diagnostic interviews and other means. [Elaboration of C.3.1]

Well-prepared beginning teachers recognize that there are many valued mathematical learning outcomes that need to be assessed. They do not focus on particular outcomes to the detriment of gaining insights into others. For instance, in a unit on geometric measurement, they assess more than students' application of learned formulas, but also find out outcomes like students' understanding of the concept of area, ability to use mathematical tools such as protractors, and attention to precision as they measure the volume of a prism. They seek to assess valued learning outcomes such as engagement in mathematical practices and mathematical dispositions, even when routes to assess them may not be straightforward.

Well-prepared beginning mathematics teachers of grades three through five utilize multiple ways to assess learning outcomes. For instance, when focusing on students' fluency with multiplication facts, they know that timed tests are not the only, first, or best, approach. They recognize that there are several components of fluency and that timed tests do not support the ability to assess strategy use, efficiency or flexibility. They identify that they may get a sense of students' accuracy – but primarily accuracy with the variable of speed. Well-prepared beginning teachers are fully aware of the negative outcomes of timed tests which include a movement away from number sense and mental computation and move toward the planting of a seed of a negative attitude toward the study of mathematics.

Instead the well-prepared beginning teacher appreciates the value at grade three through five of looking at individual performance on assessments that pinpoint the strengths of students who are struggling (2 grades below their peers). By the use of diagnostic interviews and other more individualized assessments of students' thinking, they can find the gaps in foundational knowledge from previous grades as well as position instruction at the point where students are still strong in their understanding. In this way the movement forward is not in fits and leaps (as would be with a more gross measure of student performance in a large scale assessment) but would be targeted to specific needs and built on sound footing from the learner's perspective.

Standard C.3. Knowledge of Students as Learners of Mathematics

Effective teachers understand how students' mathematical ideas develop and how to apply such understandings to every aspect of teaching. There is much to learn about students' mathematical thinking, their engagement in mathematical practices, and their mathematical dispositions. The following section elaborates on the knowledge and pedagogical practices specific to the teachers for grades three through five.

Support Students' Sense-Making

Well-prepared beginning grades three through five teachers of mathematics nurture children's proficiency with, and sense-making of, mathematical ideas, processes, and practices. They draw on knowledge gathered through assessments of children's mathematical ideas and dispositions, as well as knowledge from research of how mathematical ideas, as well as knowledge from research of how mathematical ideas develop over time. [Elaboration of C.3.1 and C.3.2]

Well-prepared beginning teachers of mathematics have foundational understandings of students' mathematical knowledge, skills, and dispositions. They also know how these understandings can contribute to effective teaching and are committed to expand and deepen their knowledge of students' as learners of mathematics.

Indicators include:

- C.3.1. Students' thinking about mathematics content
- C.3.2. Students' engagement in mathematical practices
- C.3.3. Students' mathematical dispositions

To be well-prepared for teaching mathematics in grades three through five, beginning teachers are ready to support students in developing increasingly sophisticated, and at times more abstract, notions of mathematical ideas. Students in this grade-band are building on their additive thinking toward the more sophisticated multiplicative thinking. Students explore the nature of fractions and decimals, as well as operations involving them. They build on insights from their work with whole numbers and at times must try to avoid overgeneralize lessons learned. They notice and describe more complex properties of shapes and can express measurements of shapes in multiple ways, including algebraic formulas. This requires that beginning teachers know how mathematical ideas can progress, often drawing on knowledge from research on learning progressions. They realize that using concrete, semi concrete, and abstract representations (CSA) will involve overlap and integration and the need to revisit or directly try out ideas even as abstractions are generated. They help students understand the logic that makes procedures meaningful and the power and elegance of mathematical conventions hold for working collectively on mathematics. This requires that beginning teachers actively monitor the evolution of students' ideas, be aware of likely misconceptions and open to understanding unique ways that students might express characteristics and generalizations. Beginning teachers can tailor instruction in ways that build on what students understand and consistently encourage students to stretch their mathematical thinking such as their willingness to make conjectures or thickly describe mathematics ideas and objects.

Well-prepared beginning teachers in this grade-band help student become more fluent with mathematical ideas, knowing that fluency can free mental space needed to grapple with more complex mathematical ideas. One commonly held goal of teachers in grades three through five is to support fluency with "basic facts". Beginning teachers are sensitive to the negative impacts of practices commonly used to enhance fluency, such as timed tests. Fluency should not be developed at the expense of sense-making or productive dispositions toward mathematics. There are methods of supporting students' need to make sense of ideas while also developing greater fluency and proficiency.

Well-prepared beginning teachers in this grade-band nurture personal and public engagement in mathematical practices. Well-prepared beginning teachers understand that students' mathematical identities and perceptions of mathematical status are likely to influence their participation, so they consistently work to engage all students in mathematical discussions involving explanation and critique, as well as encourage the belief that each student can all make valued contributions. Well-prepared beginning teachers know that students in grades three through five can engage meaningfully in mathematical practices and to build these practices on the foundations that students bring. For instance, students may believe that mathematical facts are established by the teacher, the textbook, "smart" classmates, or even popular will (i.e., by voting). Beginning teachers help students unpack the limitations of these notions while also engaging students in more robust forms of argument and proof.

Well-prepared beginning teachers in grades three through five are attuned to the mathematical dispositions of their students. This grade band can be one where students feel greater empowerment to investigate mathematical ideas and independently address mathematical problems. Unfortunately, this grade band can also be a time when students shift to seeing mathematics as a collection of rules that expire, something that they are not good at, and/or something in which they choose not to invest. Beginning teachers make it their business to be keenly aware of these developments and have ways of nurturing productive dispositions toward mathematics. These teachers explicitly acknowledge a student's expression of frustration with a particular idea or practice and use it as an opportunity both to help the student understand that this feeling can be a part of the learning process and support the student in learning ways to overcome challenges through strategy, effort, and engagement.

Well-prepared beginning teachers of grades three through five students know that their students can more skillfully and reflectively engage in mathematical work. As a result, they establish routines and provide students with tools that they can use to assess their mathematical thinking and the mathematical products that they produce. This goes well beyond directives to "check your work". Well-prepared beginning teachers help students express their questions and fine-tune their resources for help. They work with students to develop a shared sense of the components of high quality work, such as a graph or an explanation. They help students understand that accuracy and speed are not the only, and often not the best, measures of quality. They build a classroom culture where students can provide mathematically useful feedback to their peers and where the teacher and the textbook are not viewed as *the only sources* of mathematical validation.

Standard C.4. Social Contexts of Mathematics Teaching and Learning

Effective teachers connect with students and their families. They build on students' ways of knowing and learning, and attend to students' culture, race/ethnicity, language, gender, socioeconomic status, abilities, and personal interests. The following section elaborates on the knowledge about social contexts of mathematics teaching and learning specific to the teachers for grades three through five.

Become Ethical Advocates for Children

Well-prepared beginning grades three through five teachers of mathematics understand their role as ethical advocates for elementary children to have access to and advance in mathematics that cultivates a positive mathematics identity, connects to children's mathematical thinking and lived experiences, build partnerships with families and communities, and work to eliminate institutional and curricular barriers to learning. [Elaboration of C.4.1]

Realizing that the social, historical, and institutional contexts of mathematics impact teaching and learning, well-prepared beginning teachers are knowledgeable about and committed to their critical role as advocates for every mathematics student.

Indicators include:

- C.4.1. Access and advancement
- C.4.2. Mathematical identities
- C.4.3. Students' mathematical strengths
- C.4.4. Power and privilege in the history of mathematics education
- C.4.5. Ethical practice for advocacy

As an ethical advocate of students, teachers in grades three through five play a crucial role in cultivating and sustaining a positive learning environment that promotes productive dispositions including positive mathematics identities of students. This includes eliciting and building on students multiple mathematical knowledge bases (Turner et al., 2012) that expands to students thinking about new mathematical domains formally introduced in the content standards like rational numbers (i.e., fractions) while simultaneously connecting those mathematical concepts to out-of-school experiences in families and communities that leverage these mathematical concepts. Lesson task examples might be investigating fraction concepts used in building a community garden; analyzing public park designs for fun, access, and safety considerations; fair sharing of resources like food and screen time; modifying a recipe for larger servings or using tools such as a tape measure for home repairs.

Well-prepared beginning teachers in grades three through five understand that, with increased conceptual knowledge and procedural fluency, students have an opportunity to critically analyze strengths and limitations of algorithms and representations some of which may come from parents and grandparents schooled in different parts of the world (Perkins & Flores, 2002). Well-prepared beginning teachers of grades three through five build on this cultural knowledge, seeking assistance from family and community members to clarify unfamiliar algorithms and possible linguistic translations when needed to build a robust understanding of the algorithms and the connections to other symbolic notations and underlying concepts.

As an ethical advocate for students in grades three through five, well-prepared beginning teachers understand that developing positive relationships and trust with families about mathematics takes time and multiple opportunities. This means effectively communicating a positive mathematical vision for their child and creating opportunities to dialogue with parents and families about mathematics learned inside and outside the classroom. Funds of knowledge surveys and family outreach activities including home visits, faith-based center collaborations, community math workshops, and community math walks

will provide rich contexts to generate tasks that excite students and customizes mathematics curriculum (Aguirre et al., 2012; Civil & Bernier, 2006).

In addition, it also means clear and respectful communication with families about their child’s learning progress. In grades three through five, state testing and other assessment indicators are often formally used to describe students’ learning progress and drive placement decisions for intervention purposes. Well-prepared beginners can provide families with a holistic picture of their child’s learning progress utilizing combinations of student work and assessment data to help develop action plans that will identify strengths and areas of growth to promote mathematical learning.

The use of standardized test scores starting in the third grade greatly shapes instruction and the ways mathematics learning is assessed and communicated to multiple stakeholders. Often computational fluency is emphasized over conceptual understanding with the introduction of timed procedural fluency tests. In addition, mathematical argumentation and complex problem solving are underemphasized given the multiple choice and short response items often reflected in standardized tests. Unfortunately, this hyper-focus on testing has exacerbated deficit language about students with the use of terms such as “low”, “below basic”, “bubble”, and “gifted” to refer to student learning. Schools are also not immune to being labeled with general terms like “failing.” As an ethical advocate for students in grades three through five, well-prepared beginning teachers are aware of this political landscape, reject the use of deficit language to describe students, and employ specific strategies in the classroom, in grade-level meetings, and during work in professional learning communities. This approach provides a more comprehensive and holistic account of student mathematical progress individually and school-wide.

Part 2: Elaborations of the Characteristics and Qualities Needed by Effective Programs Preparing Mathematics Teachers for Grades Three Through Five

This section recommends what preservice programs need to do in order to prepare their students to meet the requirements of the previous section. Standards specific to grades three through five mathematics teacher preparation programs are defined and described, along with additional commentary and examples about the general standards in Chapter 3. Programs preparing candidates to teach a broader range of grades, such as P-5, should also consider the recommendations in this document for the P-2 preparation. The other standards are listed for the sake of completeness.

Standard P.1. Establish Partnerships

Effective programs for preparing mathematics teachers have significant input and participation from all appropriate stakeholders.

Indicators include:

- P.1.1. Engage all partners productively
- P.1.2. Provide institutional support

Standard P.2. Opportunities to Learn Mathematics

A high-quality preparation program for beginning teachers of mathematics for grades three through five should provide opportunities for them to learn mathematics, focusing on the big ideas in mathematics for those grades, and also on the mathematical processes to make sense of mathematics in preparation to teach children.

Mathematical Content Preparation of Grades 3 – 5 Teachers of Mathematics

Programs that support the development of well-prepared beginning mathematics teachers of grades three through five should include coursework and other experiences focused on key mathematical ideas and skills that are pivotal in those grades. [Elaboration of P.2.1]

An effective mathematics teacher preparation program provides beginning teachers with opportunities to learn mathematics that are purposefully focused on essential big ideas across content and processes that foster a coherent understanding of mathematics for teaching.

Indicators include:

- P.2.1. Attend to mathematics content for teaching
- P.2.2. Build mathematical practices and processes
- P.2.3. Provide sustained experiences

Because well-prepared beginning teachers must have substantial mathematical knowledge and skills, as well as sound mathematical dispositions, programs must include 12 credits of coursework from a mathematics department (CBMS, 2012) and other experiences that support the development of ideas and skills that are pivotal to grades three through five teaching. For quite some time, professional organizations have called for opportunities for prospective teachers to develop mathematical perspectives on the nature of mathematics as a discipline, the evolving nature of mathematics especially given technological advances, and the nature of school mathematics (NCTM, 1991). While it is important for programs to continue to implement the recommendations of the *Mathematical Education of Teachers II* report (CBMS, 2012), programs must assure that candidates are not merely putting in “seat-time,” but instead are developing the deep understanding of these important concepts in ways that are usable in, and crucial for, effective teaching.

Given the role that grades three through five teachers play in supporting the learning of the next generation of mathematicians and mathematically astute citizens, a broad base of professional groups have argued for consistent, serious and focused work on mathematics must be part of elementary teacher preparation. It is crucial that teachers preparing for these grade-bands develop mathematical knowledge that not only spans the grade levels, but also provides opportunities to understand “big ideas” (Charles, 2005) that unify mathematics across grade-band divides.

It also is crucial that opportunities to learn mathematics are geared to the development of mathematical knowledge that is usable in teaching (Ball, Thames, & Phelps, 2008). Courses must include opportunities to engage in activities such as unpacking multiple approaches to common mathematical tasks, examining multiple representations of a particular concept, exploring mathematical ideas through real-world contexts, explaining one’s solution strategies, and taking up and checking one’s understanding of the mathematical ideas of others.

Standard P.3. Opportunities to Learn to Effectively Teach Mathematics

A high-quality preparation program for beginning teachers of mathematics for grades three through five should provide opportunities for them to learn how to teach mathematics, by participating in well-designed mathematics-specific methods courses.

Mathematics Pedagogy Preparation of Grades 3 – 5 Teachers of Mathematics

Programs that support the development of well-prepared beginning mathematics teachers in grades three through five should include coursework that supports the development of practices, techniques, and habits of mind or dispositions that support the integrity of the mathematics that is taught and the mathematical development of each student.

[Elaboration of P.3.2]

Programs should include coursework dedicated to the development of content-specific practices, techniques, and habits of mind or dispositions that support sound mathematics teaching. The goals for such courses are elaborated in section 2.2 of this document. Because teaching is an interactive and action-oriented profession, effective preparation must ensure teacher candidates practice the skillful enactment of practices and application of mathematical disposition with actual children in classroom settings. It is not enough to know about mathematics teaching or to be skillful in analyzing teaching (Lampert, 2001; Ball & Forzani, 2009). Preparation should enable the doing of mathematics teaching, in which beginning teachers integrate thought, language, representation, and action in ways that are highly attuned to what students are doing and saying in an effort to advance their mathematical learning. Foundational to this type of learning are assessments geared to provide evidence, support judgment, and direct future action related to whether and how beginning teachers are able to enact mathematical teaching practices.

Mathematics Methods Coursework for Teachers of Mathematics for Grades Three Through Five

Programs that support the development of well-prepared beginning mathematics teachers in grades three through five should include at least one mathematics methods course, or the equivalent of three semester units, focusing particularly on mathematics teaching and learning in grades three through five. [Elaboration of P.3.1, P.3.2, P.3.3, and P.3.4]

The demands of teaching and learning of mathematics in the 21st century require at least one mathematics methods course focusing on grades three through five to fully prepare new teachers with a strong foundation for success. Given the goal of educating mathematics teachers who are well-prepared, the often challenging nature of initial teaching placements, and the breadth and depth of practices and dispositions needed, programs seeking to prepare beginning mathematics teachers should provide at multiple mathematics methods courses for those seeking certification to teach broader grade bands (such as P-5 or K-8). At the same time we acknowledge that differences in the designs of teacher preparation programs will shape the ways in which this goal is achieved. As a field we need to subject

An effective mathematics teacher preparation program provides candidates with multiple opportunities to learn how to teach through mathematics-specific methods courses that integrate mathematics, knowledge of students as learners, the social contexts of mathematics teaching and learning, and practices for teaching mathematics.

Indicators include:

- P.3.1. Address deep and meaningful mathematics content knowledge
- P.3.2. Provide candidates with strong foundations of knowledge about students as mathematics learners
- P.3.3. Address the social contexts of teaching and learning
- P.3.4. Provide practice-based experiences
- P.3.5. Provide effective mathematics methods instructors

these approaches to appropriate scrutiny to ensure that they actually prepare mathematics teachers who are well prepared.

Methods courses should include opportunities for teacher candidates to do mathematics, learn about children's mathematical thinking and solution strategies, identify and integrate children's lived experiences and funds of knowledge into mathematics lessons, learn various pedagogical strategies discussed in Chapter 2, as well as how to use assessment to build on student understandings and support and extend their learning.

Standard P.4. Opportunities to Learn in Clinical Settings

A high-quality preparation program for beginning teachers of mathematics for grades three through five must include carefully designed and sequenced clinical placements with support structures for candidates.

Clinical Experiences for Grades 3 – 5 Teachers of Mathematics

Programs that support the development of well-prepared beginning mathematics teachers in grades three through five develop and systematically use a collection of clinical settings that support beginning teachers work with diverse learners, curricula, and within different institutional contexts. [Elaboration of P.4.2 and P.4.3]

The settings in which beginning teachers of grades three through five learn provide supportive contexts for developing a professional practice that integrates attention to students and their learning of mathematics content, including routine opportunities to see, engage in, and reflect on the mathematics teaching in real settings of practice. Mentor teachers who are playing roles in these grades three through five settings often are responsible for instruction across multiple subjects. It is crucial that mentors of beginning mathematics teachers routinely teach mathematics, are knowledgeable about mathematics content and pedagogy, portray productive mathematical dispositions, and open up their mathematics teaching as a welcome forum for the learning of the beginning teacher. Further, program representatives in grades three through five contexts who often are asked to provide support and feedback on the teaching of many subjects, must routinely watch the mathematics teaching of the beginning teacher as well as be knowledgeable about mathematics, and portray productive mathematical dispositions.

Settings need to provide a broad array of opportunities to learn to teach. Beginners need experiences teaching children individually, in small groups, and as a whole class; teach diverse students including teaching experiences with students from many racial, gender, linguistic, and socioeconomic backgrounds; and teach students with special needs. Beginners also need opportunities to develop ways of working with and learning from parents, families, and community members/leaders so that they

The clinical experiences of effective teacher preparation programs are guided by a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for teacher candidates to teach and to learn from their own teaching and the teaching of others.

Indicators include:

- P.4.1. Collaboratively develop and enact clinical experiences
- P.4.2. Sequence school-based experiences
- P.4.3. Experience teaching with diverse learners
- P.4.4. Recruit and support qualified mentor teachers and teacher preparation supervisors

can teach mathematics in ways that draw on students' knowledge and resources, that enhance the relevance of the mathematics that is taught, and forge productive partnerships that enhance education.

Standard P.5. Recruit and Retain Teacher Candidates

An effective mathematics teacher preparation program attracts, nurtures, and graduates high quality teachers of mathematics who are representative of diverse communities.

Indicators include:

- P.5.1. Recruit strong candidates
- P.5.2. Address diverse community needs
- P.5.3. Provide experiences and support structures
- P.5.4. Assess recruitment and retention data

Closing Remarks

Well-prepared beginning teachers of mathematics for grades three through five must understand mathematics and use mathematical practices and processes, develop strong mathematical dispositions, and use mathematical tools and technology. They must learn how to plan and implement effective instruction, analyze their teaching practice, and collaborate with colleagues, families, and community members. Well-prepared beginners must understand students' mathematical thinking, their use of strategies and mathematical practices, and the development of their mathematical dispositions. Candidates must understand and be committed to an advocacy role for every mathematics learner.

Effective teacher preparation programs to prepare teachers to teach mathematics in grades three through five must develop candidates' abilities to use high-leverage, effective mathematics teaching practices (NCTM, 2014), which requires a deep understanding of the mathematics they expected to teach. Programs must include carefully designed opportunities for candidates to learn effective mathematics-specific pedagogy, to learn about children as mathematics learners, and to participate in practice-based clinical experiences that are carefully designed and sequenced.

High-quality programs designed based on these standards will support beginning teachers' growth toward teaching excellence.

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CHAPTER 6. ELABORATIONS OF THE STANDARDS FOR THE PREPARATION OF MIDDLE-LEVEL TEACHERS OF MATHEMATICS

This chapter puts forth elaborations and examples of the standards in Chapter 2, describing the knowledge, skills, dispositions, and actions that well-prepared beginning middle-level mathematics teachers need to develop, followed by elaborations and examples of the standards in Chapter 3, describing what middle-level preservice programs need to do to ensure the effective preparation of their candidates. We include in this population those seeking certification at the middle-level, as well as those seeking certification that *includes* the middle level grades including K-8, K-12 special education, K-12 ESL education, and 7-12 teacher preparation programs.

Having a coherent, well-articulated mathematics curriculum across grades PreK-12 requires middle-level mathematics teachers who are not only knowledgeable about the mathematics they are teaching, but also about the mathematical content that is developed prior to and following the middle-level years (typically Grades 6-8). In the middle school years, students transition from number to number systems, experience mathematical ideas more abstractly, and develop foundational ideas related to algebra and geometry that are explored further in high school, college and careers. The middle-level years should not be defined as getting students ready for high school, but rather defined by coherent, relevant, meaningful experiences that develop competence and confidence in every middle-level learner. Thus, well-prepared middle-level mathematics teachers should reflect the skills and dispositions outlined in the Association for Middle Level Education (AMLE) Middle Level Teacher Preparation Standards (2012), the association that collaborates with NCTM, AMTE, and CAEP in middle school teacher preparation and accreditation. Table 6.1 provides a list of the middle-level elaborations of the standards articulated in Chapters 2 and 3.

Table 6.1. Elaborations of Candidate and Program Standards for Middle-level Teachers of Mathematics

Part 1: Candidate Knowledge, Skills, and Dispositions
Mathematics needed to teach middle school <i>Well-prepared beginning teachers of middle-level mathematics understand K-12 mathematics, with deeper understanding of the content taught at the middle-level. [Elaboration of C.1.1]</i>
Mathematical Practices and Processes <i>Well-prepared beginning middle-level teachers of mathematics comprehend and demonstrate mathematical proficiency across middle-level content. [Elaboration of C.1.2]</i>
Content Progressions for Middle-level Learners <i>Well-prepared beginning middle-level teachers of mathematics have an understanding of content progressions and the way in which students develop mathematical content over time. [Elaboration of C.1.4]</i>

Strategies and Collaborations to Support Early Adolescents

Well-prepared beginning middle-level teachers of mathematics use strategies to support a range of early adolescent learners and engage other educational professionals within their setting to support student learning. [Elaboration of C.2.1 & C.2.5]

Select and Implement Meaningful and Interdisciplinary Contexts

Well-prepared beginning middle-level teachers of mathematics understand how to engage in meaningful and interdisciplinary contexts, including the use of mathematical modeling. [Elaboration of C.2.2]

Mathematical Practices of Middle-level Learners

Well-prepared beginning middle-level teachers of mathematics support emerging mathematical practices of middle-level learners. [Elaboration of C.3.2]

Responding to the Needs of Early Adolescents

Well-prepared beginning middle-level teachers of mathematics understand the developmental needs of early adolescents and use this knowledge to create and implement culturally-relevant mathematical experiences for their students. [Elaboration of C.4.3]

Equitable Structures and Systems in Middle Schools

Well-prepared middle-level teachers of mathematics have an awareness of structures in schools and systems that support and inhibit opportunities for learning. [Elaboration of C.4.1 & C.4.4 & C.4.5]

Part 2: Program Characteristics and Qualities

Content Preparation for Middle-level Teachers of Mathematics

Programs that support the development of well-prepared beginning middle-level teachers of mathematics include content preparation aligned with The Mathematical Education of Teachers 2 (MET2) and Statistical Education of Teachers (SET). [Elaboration of P.2.1 and P.2.2 and P.2.3]

Program Opportunities beyond Middle-level Mathematics

Programs that support the development of well-prepared beginning middle-level teachers of mathematics include coursework focused specifically on teaching middle-level mathematics, the middle-level learner, and learning prior to and following middle school. [Elaboration of P.3.1, P.3.2, P.3.3, and P.3.4]

Clinical Experiences in Middle-level Settings

Programs that support the development of well-prepared beginning middle-level teachers of mathematics include clinical experiences in middle schools that are exemplar sites, reflecting standards for mathematics and middle-level education. [Elaboration of P.4.1]

Part 1: Elaborations of the Knowledge, Skills and Dispositions Needed by Well-prepared Beginning Middle-level Mathematics Teachers

This section provides additional detail, commentary, and examples of the knowledge, skills, and dispositions well-prepared middle-level mathematics teachers should have, organized by the general standards described in Chapter 2.

Standard C.1. Knowledge of Mathematics for Teaching

Middle-level teachers need strong conceptual understanding, procedural fluency, and factual knowledge of the mathematics studied in elementary, middle and high schools, and an understanding of how to develop middle-level content. Middle schools often offer a range of courses, including those that offer review of elementary content and those that offer high school course content (e.g., algebra and geometry). Additionally, middle school mathematics has a strong focus on fluency with rational numbers and algebraic thinking, which has traditionally been taught via a narrow school mathematics curriculum that emphasized rote procedures, rather than mathematical practices and processes. The content and practices related to the well-prepared beginning teacher of middle-level mathematics are described in three different elaborations in this section.

Essential Understandings of Mathematics Needed to Teach at the Middle-level

Well-prepared beginning teachers of middle-level mathematics understand K-12 mathematics, with deeper understanding of the content taught at the middle-level. [Elaboration of C.1.1]

Well-prepared beginning teachers of mathematics possess appropriate mathematical knowledge of and skill in mathematics needed for teaching. They engage in appropriate mathematical practices and support their students in doing the same. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students' mathematical productions.

Indicators include:

- C.1.1. Core content knowledge
- C.1.2. Mathematical practices and processes
- C.1.3. Mathematical dispositions
- C.1.4. Analyzing the mathematical content of curriculum
- C.1.5. Mathematical knowledge to analyze mathematical thinking
- C.1.6. Using mathematical tools and technology

In the summaries below, we describe the significant concepts that a well-prepared beginning teacher of mathematics must know in order to be able to support middle-level learners. These ideas connect to and reflect the MET2 content expectations and therefore, we include the related MET2 content expectations in each section.

Ratios and Proportional Reasoning: Well-prepared beginning teachers of mathematics have the following skills and dispositions: (1) understand a ratio as a distinct entity representing a relationship different from the quantities it compares, (2) recognize the difference between proportional and non-proportional situations; (3) experience problem situations involving more than one variable with both direct and inverse variation; and (4) know and use a variety of strategies to solve problems involving ratios and proportions (Lamon, 2012; Conference Board of Mathematical Sciences (CBMS), 2012). Well-prepared beginners understand the content that precedes ratios and proportions (multiplicative comparisons and fractions) and content that follows ratios and proportions (linear functions). They are able to select tasks that lead to the use of multiple approaches, connect to relevant contexts (e.g., financial literacy), and connect to other content within mathematics (e.g., geometry and algebra). Well-prepared beginners therefore encourage students to employ a range of reasoning strategies, including

rates and scaling, ratio tables, tape or strip diagrams, double number line diagrams, and equations (proportions) (Ercole, Frantz, & Ashline, 2011; Olson, Olson, & Slovin, 2015).

Figure 6.1. MET2: Essential Ideas for Ratios and Proportional Reasoning

Middle-level mathematics teacher essential mathematical knowledge:

- Reasoning about how quantities vary together in a proportional relationship, using tables, double number lines, and tape diagrams as supports.
- Distinguishing proportional relationships from other relationships, such as additive relationships and inversely proportional relationships.
- Using unit rates to solve problems and to formulate equations for proportional relationships.
- Recognizing that unit rates make connections with prior learning by connecting ratios to fractions.
- Viewing the concept of proportional relationship as an intellectual precursor and key example of a linear relationship.

The Number System: Well-prepared beginning middle-level teachers of mathematics have a unified understanding of the number system (an expected outcome for middle school students in the CCSS-M). This includes an understanding of number and the ordering of numbers to the system of rational numbers, recognizing fractions, decimal fractions, and percents as different representations of rational numbers. They understand algorithms, visual representations, and context-based problems associated with rational number operations. They are able to analyze students' algorithms and representations to provide feedback that connects conceptual and procedural knowledge. Well-prepared beginning teachers of mathematics understand the properties of the operations and know that these properties provide access to efficient or novel solution strategies.

Figure 6.2. MET2: Essential Ideas for The Number System

Middle-level mathematics teacher essential mathematical knowledge:

- Understanding and explaining methods of calculating products and quotients of fraction, by using area models, tape diagrams, and double number lines, and by reading relationships of quantities from equations.
- Using properties of operations (the CCSS term for the field axioms) to explain operations with rational numbers (including negative integers).
- Examining the concepts of greatest common factor and least common multiple.
- Using the standard U.S. division algorithm to explain why decimal expansions of fractions eventually repeat and showing how decimals that eventual repeat can be expressed as fractions.
- Explaining why irrational numbers are needed and how the number system expands from rational to real numbers.

Algebraic Thinking and Functions: Well-prepared beginning middle-level teachers of mathematics have a strong understanding of algebraic thinking, noticing the central role of generalization and the use of

variables to represent numbers. In particular, they understand algebraic thinking as (1) the study of structures in the number system, including those arising in arithmetic; (2) the study of patterns, relations, and functions; and (3) the process of mathematical modeling (Kaput, 2008; Lloyd, Herbel-Eisenmann, Star, & Zbiek, 2011). Well-prepared beginners at the middle-level are aware that symbols such as equal signs, inequality symbols, and square root symbols can be difficult for students and that connecting these symbols to their meanings is central to success in algebra. For example, they may ask questions, such as, “what answers make sense *given the context* of this problem?” or, “does a solution set containing only the number 0 indicate having a solution?” They understand the critical importance of equivalence and approach the teaching of algebraic concepts by explicitly attending to equivalence, for example, asking students to justify methods for simplifying expressions by justifying that the new form is equivalent.

Well-prepared beginners have a strong conceptual and procedural understanding of linear equations, systems of linear equations, linear functions, and slope of a line. They understand that functions describe a relationship or situation where one quantity determines another and are able to help learners understand the meaning of function and use appropriate notations and representations. Well-prepared beginners have a strong understanding of how different mathematical representations such as graphs, tables, and equations support and influence algebraic thinking and functional thinking. They have strategies for integrating representations into instruction in ways that help students move flexibly among different representations and connect each representation to the given context/situation.

Figure 6.3. MET2: Essential Ideas for Algebraic Thinking and Functions

Middle-level mathematics teacher essential mathematical knowledge:

- Viewing numerical and algebraic expressions as “calculation recipes,” describing them in words, parsing them into their component parts, and interpreting the components in terms of a context.
- Examining lines of reasoning used to solve equations and systems of equations.
- Viewing proportional relationships and arithmetic sequences as special cases of linear relationships. Reasoning about similar triangles to develop the equation $y = mx + b$ for (non-vertical) lines.
- Examining and reasoning about functional relationships represented using tables, graphs, equations, and descriptions of functions in words. In particular, examining how the way two quantities change together is reflected in a table, graph, and equation.
- Examining the patterns of change in proportional, linear, inversely proportional, quadratic, and exponential functions, and the types of real-world relationships these functions can model

Geometry and Measurement: Well-prepared beginning teachers of mathematics see the value of geometry for middle-level learners, are able to connect geometry to measurement and algebra to showcase the importance of geometry across the content while understanding that geometry itself is relevant and important for middle-level learners (Sinclair, Pimm, & Skelin, 2012). They have strategies for connecting geometry to ratios, proportions, and algebraic thinking as they explore scale drawings and transformations. Another critical connection between algebra and geometry important to middle grades mathematics is the Pythagorean Theorem; well-prepared beginners can explain why the Pythagorean Theorem is true (e.g., by decomposing a square in two different ways), apply the

Pythagorean Theorem, and select high quality tasks and facilitate lessons in which their students are also able to explain and use the Pythagorean Theorem.

Well-prepared beginning teachers of mathematics understand that measurement is an important and useful mathematics content strand and therefore should be applied to authentic, culturally relevant contexts. Additionally, measurements are interrelated and must be understood and taught in ways that showcase the connections, such as connections within area formulas, and connections among length, area, and volume concepts and formulas.

Figure 6.4. MET2: Essential Ideas for Geometry and Measurement

Middle-level mathematics teacher essential mathematical knowledge:

- Deriving area formulas such as the formulas for areas of triangles and parallelograms, considering the different height–base cases (including the “very oblique” case where “the height is not directly over the base”).
- Explaining why the Pythagorean Theorem is valid in multiple ways. Applying the converse of the Pythagorean Theorem.
- Informally explaining and proving theorems about angles; solving problems about angle relationships.
- Examining dilations, translations, rotations, and reflections, and combinations of these.
- Understanding congruence in terms of translations, rotations, and reflections; and similarity in terms of translations, rotations, reflections, and dilations; solving problems involving congruence and similarity in multiple ways.

Statistics and Probability: A well-prepared beginning teacher of mathematics should have experience with the important process components explained in the American Statistical Association (ASA) Guidelines for Assessment and Instruction in Statistics Education such as formulating questions, collecting and analyzing data, and interpreting results (ASA, 2007). They are committed and able to engage students in statistical thinking and data analysis processes beyond computation of values, calculation of statistical measures, and rote creation of data displays that draws focus away from in-depth analysis. They have a solid understanding of variability, and are able to describe the center and spread of data, understand which measures might be used in situations, and engage students in the selection of measures to describe data. They understand, and help students understand, that different types of graphs and other data representations provide different information about the data and, therefore the choice of graphical representation can affect how well the data are understood. Well-prepared beginners have a strong understanding of bivariate data, how to represent it, and how to help students see the connections to proportional and algebraic reasoning.

Well-prepared beginners have a deep understanding of chance and the misconceptions that people have about chance (e.g., the chance occurrence of 5 heads on a coin toss has no effect on whether another head will occur on the next coin toss). They appreciate that experiments, including simulations can help students understand probability concepts, as well as understand how probability is used in real-life contexts beyond games. Additionally, a well-prepared beginning teacher knows how to build connections between number and geometry to probability.

Figure 6.5. MET2: Essential Ideas for Statistics and Probability

Middle-level mathematics teacher essential mathematical knowledge:

- Understanding various ways to summarize, describe, and compare distributions of numerical data in terms of shape, center, and spread.
- Calculating theoretical and experimental probabilities of simple and compound events, and understanding why their values may differ for a given event in a particular experimental situation.
- Developing an understanding of statistical variability and its sources, and the role of randomness in statistical inference.
- Exploring relationships between two variables by studying patterns in bivariate data.

Mathematical Practices and Processes Needed to Teach Middle School.

Well-prepared beginning middle-level teachers of mathematics comprehend and demonstrate mathematical practices and processes across middle-level content. [Elaboration of 2.1.2]

Well-prepared beginning middle-level teachers of mathematics must be able to demonstrate mathematical proficiency across the middle-level content areas described in Standard 6.1 as they are learning (or re-learning) the mathematics. This means that they are able to model such things as considering various options for solving a problem, selecting an efficient strategy given the numbers or variables in the problem, using appropriate representations, models, and tools, and noticing underlying structures or patterns that can provide insights into solving a problem. Additionally, well-prepared beginners know what mathematical practices or processes are described in their state's middle-level mathematics standards (e.g., the Mathematical Practices in the CCSS-M) and that these standards are reflected in the middle-level content, not additional, discrete topics.

Although beginning teachers of mathematics will not see all connections between mathematical practices and content, well-prepared beginners are able to articulate strong connections between practices and content for at least one major middle school concept, as well as describe some connections across the middle school topics described in Standard 6.1. For example, they understand the importance of contextualizing and decontextualizing (CCSS-M Mathematical Practice 2) with regard to mathematical problems presented through authentic situations. They apply that to content instruction, such as fraction division, through using contextualized settings in which they have students explore the ways in which solution strategies or procedures can be derived.

Well-prepared beginners are aware of physical and technological tools that can support learning of middle-level mathematics. Selecting appropriate tools is a critical mathematical practice for middle-level learners, as they transition from more concrete content to more abstract mathematical representations. Technology tools allow teachers and students to connect different representations of mathematical concepts and build learner knowledge within and among different representations (NCTM, 2014). Algebraic thinking, for example, includes significant use of equations, tables, and graphs. They can use virtual tools such as graphing utilities including data graphing tools, dynamic geometry software, and electronic spreadsheets as well as physical models and tools, in various ways (e.g., for enrichment, whole class instruction, review, new instruction, etc.) to develop students' knowledge and understanding of mathematics.

Content Progressions for Middle-level Learners

Well-prepared beginning middle-level teachers of mathematics have an understanding of content progressions and the way in which students develop mathematical content over time. [Elaboration of C.1.4]

Understanding how content builds on other content is a critical component of a middle-level teachers' content knowledge. As discussed in Chapter 2, available content progressions provide considerably more detail on how content develops in sophistication over time than is apparent in most standards documents. Well-prepared beginners of middle-level mathematics recognize that content progressions and learning trajectories are important resources, while also recognizing that each learner is unique, with different prior knowledge and different ways they might approach solving problems and doing mathematics. For example, the progression of proportional reasoning is a critical component to middle-level mathematics education. Ratios are grounded in multiplicative comparisons learned in elementary school. In middle school, students explore unitizing and rates. There are many ways that students may reason about ratios and rates, from informal strategies to the use of diagrams, ratio tables, graphs, and equations, and these ways of reasoning may be what personally makes sense to them or reflect they way they learned about multiplicative comparisons or ratios previously or at home. A well-prepared beginner is able to determine and support the individual ways students' reason about ratios and rates, while also helping each student deepen their knowledge by understanding other ways of solving ratio and proportion problems. This solid foundation then leads to developing the concept of slope as based on ratio and proportional reasoning with respect to linearity, which similarly can be conceptualized in a variety of ways. Valuing different ways of thinking about ratios, rates, proportions, slope, and so on, is important in developing each students' mathematical identity. Additionally, middle-level learners may feel awkward in sharing their unique thinking. While it may be beyond a beginner to have a collection of strategies for assessing each learner's' unique understandings, well-prepared beginning teachers of mathematics understand the importance of looking for students' unique mathematical reasoning and have strategies for respecting, soliciting and understanding the mathematical representations and explanations of their students. Middle-level mathematics are critical years for developing students algebraic thinking, which is then continued in high school with the study of more advanced algebraic concepts.

Standard C.2. Knowledge and Pedagogical Practices for Teaching Mathematics

Middle-level learners must see the relevance and intrigue of mathematics, including its connections to the other content they are learning. Well-prepared beginning teachers of middle-level mathematics have a beginning repertoire of student-relevant contexts for each topic they are teaching, and understand the importance of using contexts to engage students in the content. As described in the Association of Middle-Level Education Standards, beginning middle-level teachers, “facilitate relationships among content, ideas, interests, and experiences by developing and implementing relevant, challenging, integrative, and exploratory curriculum” (AMLE, 2012, Standard 2, Element c.).

Strategies and Collaborations to Support Early Adolescents

Well-prepared beginning middle-level teachers of mathematics use strategies to support a range of early adolescent learners and engage other educational professionals within their setting to support student learning. [Elaboration of C.2.1 & C.2.5]

Well-prepared beginning teachers of mathematics have a foundation of pedagogical content knowledge, effective and equitable mathematics teaching practices, and a positive and productive disposition toward teaching mathematics to support students’ sense-making, understanding, and reasoning.

Indicators include:

- C.2.1. Promoting equitable teaching
- C.2.2. Planning for effective instruction
- C.2.3. Implementing effective instruction
- C.2.4. Analyzing teaching practice
- C.2.5. Enhancing teaching through collaboration with colleagues, families, and community members

Students in the middle school years are reaching a developmental stage that involves new biological and psychological experiences that may lead to sudden changes in interests and behaviors (AMLE, 2015). Well-prepared beginners are knowledgeable about the nature and developmental needs of early adolescents. For example, as children reach early adolescence, their cognitive development in mathematics sometimes far exceeds their biological or psychological development. Well-prepared beginners look for and support good mathematical thinking, recognizing that a learner’s behaviors might make it more challenging to determine what they actually know and can do. Mathematics students in the middle grades, like other grades, also represent a spectrum of learners that include learners with extraordinary talents and gifts for mathematics and learners with cognitive or psychological disabilities. Equally important, the needs of many students may not have met in previous grades allowing them to meet their potential as learners of mathematics.

Well-prepared beginners understand the specific needs of emergent multi-lingual learners, recognizing that these needs vary greatly with each child based on various contextual factors, such as their fluency in their native language, fluency in English, familiarity with U.S. culture, their own cultural mathematical practices, and so on. Well-prepared beginners understand that there are mathematics-specific linguistic and cultural considerations in teaching middle-level mathematics, and seek specialists and other resources to ensure they are supporting and challenging each student.

Well-prepared beginners seek to and are able to recognize mathematically promising students as well as create a learning environment that helps all learners excel. They have the disposition to seek out specialists in order to support and challenge students in their classrooms, as well as suggest enrichment options beyond the classroom, such as clubs (e.g. The National Math Club, Odyssey of the Mind, Creative Adventures in Mathematics) and competitions (e.g. MATHCOUNTS, the American Mathematics Competition, Mathematical Olympiad).

Well-prepared beginners seek to support and challenge students with disabilities or learning challenges, accessing specialists to support their efforts. Because approximately 13% of all public school students receive special education services (National Center for Education Statistics, 2015) and because middle level mathematics learning requires significant mathematics expertise, well-prepared beginners value and seek to co-teach with professionals who have training for working with learners' special needs (but lack mathematical expertise).

Vignette: Co-planning and Co-teaching to Support Every Student

Context: Mr. W is a sixth grade mathematics teacher with 28 students including five with special needs. Mary, Angela, Morgan, and Richard have intellectual disabilities including difficulty with reading comprehension and Jackson is autistic and exhibits difficulty with social skills, but functions well cognitively. Ms. G, a special education teacher, co-teaches with Mr. W. Mr. W. has begun a unit that includes expressing one quantity, the dependent variable, in terms of the other quantity, the independent variable. This lesson is designed to help students reason about the relationship between two variables, using concrete situations and graphs. Students will receive cards with situations in words (e.g., "height of a ball thrown straight up into the air from the time it was thrown until it hits the ground") and cards with graphs that will be matched with the stories.

Co-Planning: The day before the lesson Mr. W and Ms. G reviewed the lesson plan and anticipated that Mary, Angela, Morgan, and Richard might struggle to read the scenarios. Jackson will likely need support discussing why he chose his graph. Mr. W anticipates that all of his students might struggle with understanding the variables that might be used for the two axes of the graphs. They decide to scaffold the activity, beginning with a whole class activity, then having students work in groups of four. Ms. G and Mr. W will be sure to monitor the learners who might struggle with reading. Mr. W will approach Jackson so that he can practice his explanation before the whole class discussion.

Co-Teaching: Mr. W begins by engaging the class in an example of a (real) ball thrown in the air. He then has students read the scenario. He asks what the variables are and what it might look like on a graph. He shows two graphs and asks students to tell why one matches and why the other one does not. During small group time, Ms. G notices that many students are having difficulty with the reading, so she encourages these groups to summarize the meaning of different scenarios, giving the gist first, and then adding details. All students are able to match scenarios to graphs. Mr. W and Ms. G take turns calling on a representative from each group to explain one scenario and the associated graph to the class and each provide support feedback and comments. Jackson (prompted to rehearse his response by Ms. G) accurately and willingly shared a rationale for a match.

Importantly, well-prepared beginners know that strategies such as multiple representations of concepts, multiple means of student action and expression, and multiple methods for engagement are particularly important to middle-level learners who are transitioning to more abstract mathematical concepts. While they may be seeking guidance from specialists for students identified for such services, they also have a disposition to continuously find, try, and evaluate their own strategies to engage, inspire, and support every child. They recognize the critical importance of relationships for middle-level learners, and seeks to establish relationships with each student so that they are better able to build on that student's strengths and interests to develop that student's mathematical skills and identity.

Meaningful and Interdisciplinary Contexts

Well-prepared beginning middle-level teachers of mathematics understand how to engage middle-level learners in meaningful and interdisciplinary contexts, including the use of mathematical modeling. [Elaboration of C.2.2]

Many contexts, interesting and accessible to middle-level learners, can be investigated using mathematics. And, many of these contexts can be connected to middle-level content in the other disciplines (science, language arts, social studies, as well as other content). Well-prepared beginners consider ways to design interdisciplinary instruction, and are able to engage in interdisciplinary conversations offering ideas for how important mathematics can be connected to other disciplines (AMLE, 2012). They distinguish between using mathematics as a computational tool and using mathematical reasoning or modeling, and seek to find meaningful connections for their students.

Well-prepared beginning teachers of middle-level mathematics are knowledgeable about context-based mathematical modeling and design-based activities for middle-level learners. Mathematical modeling not only provides opportunities for interdisciplinary instruction, it provides authentic contexts for doing mathematics and builds mathematical understanding (Hirsch, 2016; Usiskin, 2015). The modeling task in Figure 6.6 involves a problem about the consequences of a glacier melting (adapted from the Borba, Villareal, and Soares (2016), pp. 144-148). Such a task could be implemented in collaboration with teachers of English Language Arts (ELA), Social Studies, and Science instruction. The composition and production of a final report can include ELA goals, the process of persuading elected officials or members of the public can be addressed as part of social studies, and the measurement and data gathering expectations align with science standards.

Figure 6.6. Interdisciplinary Modeling Task for 6-8

Melting of a Glacier

The problem that we posed was to observe the percentage of reduction of a glacier, in this case, the Puncak Jaya glacier located in Indonesia. [. . .] this theme seemed interesting to us since, because of mankind, we are destroying the environment. Our hypothesis is that the glaciers have diminished to the point of almost disappearing. But in order to [prove] this hypothesis, we should consult diverse sources.
(adapted from Borba et. al (2016), pp. 144-148)

Well-prepared beginners recognize that how a task is implemented in a classroom influences how meaningful and engaging it is for students. Figure 6.7 describes effective teaching practices for supporting engineering design (and mathematical modeling).

Figure 6.7. Middle-Level Mathematics Teacher Preparation and Engineering Design

Excerpt from *Engineering in K-12 Education: Understanding the Status and Improving the Prospects*
(National Research Council, 2012)

With considerable teacher support, both early elementary students and middle school students can move toward a conceptual understanding that emphasizes function, just as experienced designers do (Penner et al., 1998).

Effective teacher strategies include:

- (1) pointing out limitations of the class models as a whole (e.g., if none of the initial models includes a mechanism for motion, the teacher may suggest that students consider the specific idea of motion in their revisions);
- (2) providing information when there is no way for students to discover the information on their own (e.g., providing the mathematical concept of median as a way of representing a range of data); and
- (3) encouraging individual teams of students to pursue specific design challenges that extend their models in general ways (e.g., considering how the function of the object under investigation is similar to and different from a familiar related object).

Students whose teachers used these strategies were able to design increasingly complex functional models, including models of the mechanism of motion, and then to develop data representations to support their claims about the performance of their designs. (p. 124)

It takes significant time for teachers to develop robust understandings of ways in which content can be integrated, and the ways in which engineering design and modeling activities can support mathematical content goals within any given classroom. For example, weighing the cost benefits of paint, ensuring lead does not impact safety, and considering sustainable materials all provide meaningful experiences that address science and social studies. While teachers throughout their careers will continue to learn about these relevant applications, well-prepared beginners know and have access to resources that provide engaging interdisciplinary mathematics investigations and mathematical modeling activities.

Standard C.3. Knowledge of Students as Learners of Mathematics

Middle-level students have their own unique understandings from their elementary schooling, life experiences, and personal preferences. The complex mathematical ideas at the middle-level can almost always be approached in a variety of ways. Therefore, mathematics teachers at the middle level must prioritize individual student reasoning as part of their planning and teaching.

Well-prepared beginning teachers of mathematics have foundational understandings of students' mathematical knowledge, skills, and dispositions. They also know how these understandings can contribute to effective teaching and are committed to expand and deepen their knowledge of students' as learners of mathematics.

Indicators include:

- C.3.1. Students' thinking about mathematics content
- C.3.2. Students' engagement in mathematical practices
- C.3.3. Students' mathematical dispositions

Develop Mathematical Practices of Middle-level Learners

Well-prepared beginning middle-level teachers of mathematics support emerging mathematical practices of middle-level learners. [Elaboration of C.3.2]

Early adolescents benefit from opportunities to explore meaningful and authentic tasks that relate to their interests and backgrounds (AMLE, 2012). Such authentic contexts provide an environment from which middle-level learners can further their ability to use mathematical practices and processes. As noted earlier in this chapter, middle-level mathematics is more abstract and symbolic than elementary school mathematics. Middle-level learners must have regular opportunities to engage in mathematical practices and processes in a more sophisticated manner than they may have demonstrated in earlier grades. Therefore, well-prepared beginners must be able to engage their students in representing and explaining their mathematical thinking and making mathematical arguments. For example, they must be able to understand the various approaches students might use to solve problems and create an environment in which strategies are discussed, critiqued, and compared.

The sequence of activities in Figure 6.8 (adapted from Steinhorsdottir, n.d.) is designed to develop a deeper understanding of proportional reasoning, focusing on the fact that the numbers provided in a missing value proportion problem influence the ways in which the task will be solved (by both teacher candidates and middle level students). The series of activities has mathematics teacher candidates examine their mathematical knowledge related to proportional reasoning and then consider how middle-level students might engage in the same task.

Figure 6.8. A sequence of course activities focused on developing pedagogical content knowledge

Activity 1. Solve the Potion Problem. The preservice teachers (PSTs) first solve The Potion Problem, a missing value proportion problem, on their own, in as many ways as they can, trying to consider ways in which middle-level students will approach the problem:

The Potion Problem: A potion calls for 4 drops of magical Bulbadox Juice for every 2 cat hairs. Neville squeezed the dropper too hard! If 44 drops of the magical Bulbadox Juice were in his pot, how many cat hairs should he use?

A variety of strategies are shared and compared during a whole-class discussion. The class comes to an understanding of some generalized strategies for missing value problems, including multiplicative within measure space, multiplicative between measure space, build up, additive error, and other errors (For full descriptions of these strategies, see Riehl & Steinhorsdottir, 2014). As a follow up assignment, PSTs solve two new missing value problems in which they attempt to use each of the correct strategies on both of the assigned problems.

Activity 2. Compare Problem Numbers and Related Strategy Selection. During the next class, preservice teachers discuss The Potion Problem and the two new problems assigned with respect to number choice in the problems and its impact on potential middle-level students' strategies used to solve them (i.e., the given numbers influence which strategy is selected). In the first new problem, the within measure space scale factor is a whole number while the between measures space ratio is a rational number and in the second new problem both the within measure space and between measure space comparisons yield rational numbers. Second, PSTs analyze middle-level student work from the two new problems to see to what extent the numbers in the task influenced strategy selection. Third, PSTs view a video clip of

a researcher interviewing a middle-level student with one of the problems they had just explored and discuss the student's mathematical thinking as well as the interviewer's posing of questions.

Activity 3. Interview a Middle-Level Student. Preservice teachers individually conduct an interview with a middle-level student, given a set of 15 missing value problems (with varying number choices). To prepare, they solve the problems so they are able to select the most appropriate problems to pose to their student based on what they are observing. They analyze the video-recording of the interview, considering the mathematical thinking of the middle-level student as well as their own posing of questions.

Well-prepared beginners also recognize that early adolescents may be self-conscious and therefore have a variety of ways for students to demonstrate their unique thinking. For example, a well-prepared beginner might collect work and project particular solutions anonymously to highlight reasoning or particular representations. Well-prepared beginners understand that particular teacher moves support (or inhibit) student development of mathematical practices and processes. For example, as discussed in Activities 2 and 3 in Figure 6.8, posing questions can help to develop students' abilities to analyze problem situations, select appropriate strategies, and reason quantitatively. Well-prepared beginners reflect on their own actions as they impact the ongoing development of student thinking.

Standard C.4. Social Contexts of Mathematics Teaching and Learning

As suggested by John Dewey (1910), improving the public's knowledge of mathematics and science and cultivating critical reasoning skills are important goals for education and essential for a democratic citizenry. Middle grades become an important time in a child's development to bring numerical and other mathematical reasoning tools to bear on the decisions citizens must make in a democracy even when research shows that strongly held beliefs and cultural affinities may disable individuals' use of reason and sense-making to inform decisions (Kahan, Peters, Dawson, & Slovic, 2013). Mathematics, in particular, provides young, future citizens with useful tools and practical lenses to examine social and personal issues that arise throughout their lives. Middle school mathematics teachers must communicate and model the power and utility of mathematics for decision-making and personal growth.

Realizing that the social, historical, and institutional contexts of mathematics impact teaching and learning, well-prepared beginning teachers are knowledgeable about and committed to their critical role as advocates for every mathematics student.

Indicators include:

- C.4.1. Access and advancement
- C.4.2. Mathematical identities
- C.4.3. Students' mathematical strengths
- C.4.4. Power and privilege in the history of mathematics education
- C.4.5. Ethical practice for advocacy

Responding to the Needs of Early Adolescents

Well-prepared beginning middle-level teachers of mathematics understand the developmental needs of early adolescents and use this knowledge to create and implement culturally-relevant mathematical experiences for their students. [Elaboration of C.4.3]

Well-prepared beginners at the middle-level are able to cultivate the development of students' positive mathematical identities and draw on students' cultural and linguistic strengths as well as their individual

interests and passion as part of the middle school mathematics instructional program. As an example, using personal information surveys can help early adolescent learners to examine data and measurement units that are plausible (Lovett & Lee, 2016). Similarly, proportional reasoning skills can be developed by examining socially relevant questions about equity and fairness in the contexts of consumer issues, population growth, and crime rates (Simic-Muller, 2015). Drawing on how different cultures employ mathematical ideas provides students an opportunity to learn about and honor their own and other cultures as exemplified in how students from immigrant families might participate in “a culture-laden lesson” that develops “a compassionate understanding of their classmates from different backgrounds and [fosters] an atmosphere of respect, solidarity, and collaboration” (Taylor, Rehm, & Catepillán, 2015).

Further, the middle school mathematics classroom can be a place where early adolescent learners examine the many difficult and complicated challenges they will confront at school and in their lives using the analytical and logical tools provided by mathematics. Well-prepared beginners must be careful to avoid advocacy of a particular point of view, but can examine how mathematics informs opinions and decisions about topics that are current, relevant, or of particular interest to learners (such as climate and the environment, health and human sexuality, bullying, or lotteries) and thereby empowering students to use mathematics for critical thinking, reasoning, and sense making. Beginners realize that such careful decision making and enactment of such a curriculum design shapes students emerging mathematical identities and influences the decisions they will make in terms of continuing in mathematics, pursuing careers, and selecting college majors.

Equitable Structures and Systems in Middle Schools

Well-prepared middle-level teachers of mathematics have an awareness of structures in schools and systems that support and inhibit opportunities for learning. [Elaboration of C.4.1 & C.4.4 & C.4.5]

Tracking, the practice of grouping students in different classes based on perceived ability levels, is a particular structure in schools that often leads to students being enrolled in algebra courses early in the middle grades, and other students kept from enrolling in such courses. This practice is typically introduced in the middle grades (Loveless, 2016), and usually involves earlier than high school placement in a first year algebra course. Tracking has been demonstrated to create and reinforce social inequities because African-American, Latin@ and children living in poverty are underrepresented in the accelerated tracks (Boaler, 2011; Larnell, 2016).

Enrichment and acceleration are two different things. Enrichment provides opportunities within courses to deepen student understanding; acceleration is a faster pace through a curriculum. Both are used in middle-level to differentiate instruction, and yet these strategies have resulted in inequities and denied students access to important mathematics and resulted in too many students not being college and/or career ready.

This particular phenomenon of rushing elementary learners to algebra and high school learners to calculus is generally identified as acceleration, and is counter to ample research (Bressoud, Mesa, & Rasmussen, 2015). This phenomenon has contributed to practices at the middle level that are inconsistent with the principles of middle level education articulated by the Association for Middle Level Education (AMLE, 2010), and runs counter to principles of equity. Accelerating curriculum, such as Algebra I being an option in 8th grade, can be done equitably and fairly, but decisions based solely on high stakes test scores or other proxy measures of student ability are likely to negatively impact a large segment of the middle school student population that might otherwise develop positive mathematical identities (See C.4.2 for additional discussion). When acceleration in algebra is an option, for example,

placement decisions must be based on multiple measures and not discriminatory. Furthermore, all courses must provide enriched curriculum at the student’s grade level. As noted by Gojak (2013), middle grades can be considered to be a time to get “messy with mathematics.” Viewed through the lens of enrichment, traditional discussions of acceleration become less important than developing ways in which all middle grades students’ mathematical experiences can become more enriched.

Part 2: Elaborations of the Characteristics and Qualities Needed by Effective Programs Preparing Middle-level Teachers

This section provides additional detail, commentary, and examples what preservice programs need to do in order to effectively prepare their students to teach middle-level mathematics, organized by the general standards described in Chapter 3.

Standard P.1. Establish Partnerships

Partnerships are particularly important in ensuring the effective preparation of middle-level mathematics teachers. Partnering with middle schools that model middle-level practices provides opportunities for candidates to experience the benefits of teaming and interdisciplinary instruction, among other things. Partnerships between colleges of education and departments of mathematics, for example, provide an opportunity to coordinate the content, experiences, and sequencing of middle-level mathematics content, mathematics methods, and other middle-level courses. Chapter 3 provides significant guidance on building effective partnerships that can support the preparation of well-prepared beginning teachers of mathematics at the middle-level.

Effective programs for preparing mathematics teachers have significant input and participation from all appropriate stakeholders.

Indicators include:

- P.1.1. Engage all partners productively
- P.1.2. Provide institutional support

Standard P.2. Opportunities to Learn Mathematics

Well-designed programs include content courses that address these content needs of middle-level mathematics teachers. As described in Part 1 of this chapter, middle-level candidates must be prepared to teach a broad range of mathematics topics, understanding the content, progressions, relationships among content, not just at the middle level, but before and after middle-level. Further, middle-level candidates need to understand historical and cultural aspects of mathematics. Regardless of licensure options (e.g., K-8, 5-8, 6-12, or any other) effective preparation of middle-level mathematics teachers include opportunities to learn deeply the mathematics content that is taught at the middle-level, as well as mathematics content that comes before and after.

An effective mathematics teacher preparation program provides beginning teachers with opportunities to learn mathematics that are purposefully focused on essential big ideas across content and processes that foster a coherent understanding of mathematics for teaching.

Indicators include:

- P.2.1. Attend to mathematics content for teaching
- P.2.2. Build mathematical practices and processes
- P.2.3. Provide sustained experiences

Content Preparation for Middle-level Teachers of Mathematics.

Programs that support the development of well-prepared beginning middle-level teachers of mathematics include content preparation aligned with The Mathematical Education of Teachers 2 (MET2) and Statistical Education of Teachers (SET). [Elaboration of P.2.1 and P.2.2 and P.2.3]

Consistent with the recommendations of MET II and SET, the following coursework is needed to prepare middle-level teachers:

- At least 15 semester-hours (or equivalent) of mathematics and statistics courses designed specifically for future middle-level teachers, including courses that engage middle-level candidates in opportunities to demonstrate the mathematical practices.
- At least 9 semester-hours (or equivalent) of mathematics and statistics courses beyond the precalculus level, including at least one statistics course.

Importantly, the content within these courses must address the content needs of middle-level mathematics teachers. Across the 24+ hours of coursework, a high quality middle-level mathematics preparation program provides coursework that addresses the following mathematical concepts (content lists and course recommendations are from MET2 (CBMS, 2012)) and SET (Franklin, et al., 2015):

Number and Operations. Number and operations in base ten, fractions, addition, subtraction, multiplication, and division with whole numbers, decimals, fractions, and negative numbers. Possible additional topics are irrational numbers or arithmetic in bases other than ten. (Recommendation: 6 semester-hours)

Geometry and Measurement. Perimeter, area, surface area, volume, and angle; geometric shapes, transformations, dilations, symmetry, congruence, similarity; and the Pythagorean Theorem and its converse. (Recommendation: 3 semester-hours)

Algebra and Number Theory. Expressions and equations, ratio and proportional relationships (and inversely proportional relationships), arithmetic and geometric sequences, functions (linear, quadratic, and exponential), factors and multiples (including greatest common factor and least common multiple), prime numbers and the Fundamental Theorem of Arithmetic, divisibility tests, rational versus irrational numbers. Additional possible topics for teachers who have already studied the above topics in depth and from a teacher's perspective are: polynomial algebra, the division algorithm and the Euclidean algorithm, modular arithmetic. (Recommendation: 3 semester-hours)

Statistics and Probability. Describing and comparing data distributions for both categorical and numerical data, exploring bivariate relationships, exploring elementary probability, and using random sampling as a basis for informal inference. An effective program requires not only an introduction to statistics, but a course for middle level teachers emphasizing data collection and analysis. Such an experience emphasizes active learning with appropriate hands-on devices and technology while probing deeply into the topics taught in the middle grades, all built around seeing statistics as a four-step investigative process involving question development, data production, data analysis and contextual conclusions (ASA/NCTM, 2015; CBMS, 2012; Franklin, et al., 2015). (Recommendation: 6 semester-hours)

Implicit in these lists is the importance of understanding the mathematical content in elementary as well as high school levels. For example, one of the two course recommendations in the area of number, may be a mathematics course for elementary teachers focused on rational numbers. Additionally, a well-

designed middle-level mathematics program has strategically considered the sequence in which courses occur, including the extent to which courses are taken prior to or concurrent with education courses, mathematics methods courses, and clinical experiences. While having mathematical knowledge is a prerequisite to teaching mathematics, having mathematics course opportunities later in a program can have more connection to a middle-level candidates classroom teaching. A well-designed program provides at least one content course experience that is concurrent to a candidate's clinical experience so that connections can be made between the content being learned and content that is being taught in the clinical setting.

Standard P.3. Provide Opportunities to Learn to Teach Mathematics

As described in chapter 3, mathematics methods courses are critical to the preparation of well-prepared beginners. In order for middle-level teacher candidates to be well-prepared, they need middle-level focused methods courses where they will have the opportunity to apply their developing teaching skills and their knowledge of mathematics to the teaching of specific mathematical topics at the middle level. This is a required, but not sufficient preparation for a well-prepared middle-level candidate. A high quality program also requires coursework that prepares the middle-level teacher of mathematics to understand the learner and the content they have learned prior to middle school and what they will learn after middle school.

Program Opportunities Beyond Middle-level Mathematics

Programs that support the development of well-prepared beginning middle-level teachers of mathematics include coursework focused specifically on teaching middle-level mathematics, the middle-level learner, and learning prior to and following middle school. [Elaboration of P.3.1, P.3.2, P.3.3, and P.3.4]

An effective mathematics teacher preparation program provides candidates with multiple opportunities to learn how to teach through mathematics-specific methods courses that integrate mathematics, knowledge of students as learners, the social contexts of mathematics teaching and learning, and practices for teaching mathematics.

Indicators include:

- P.3.1. Address deep and meaningful mathematics content knowledge
- P.3.2. Provide candidates with strong foundations of knowledge about students as mathematics learners
- P.3.3. Address the social contexts of teaching and learning.
- P.3.4. Provide practice-based experiences
- P.3.5. Provide effective mathematics methods instructors

In order for middle-level teacher candidates to be well-prepared, they need middle-level focused methods courses where they will have the opportunity to apply their developing teaching skills and their knowledge of mathematics to the teaching of specific mathematical topics at the middle level. In addition to this mathematics-specific, middle-level experience, those preparing to teach middle level mathematics should have at least one additional intense learning opportunity focused on the middle level learner. This may be a general middle-level course, or a second content methods course (e.g., a second mathematics methods or a science methods). Having more than one course focused on middle-level learners or a second content area provide candidates with the opportunity to explore interdisciplinary connections, as well as other aspects of the middle school and middle-level learners, integrative and experiential curriculum, instructional strategies appropriate to the early adolescent, and the way in which middle schools function.

As the name of the grade band indicates, middle-level educators are at a transitional point in a learner's K-12 mathematics education. Additionally, mathematics experiences and learning are transitioning across these years from very concrete and visual content towards much more abstract and complex content. Students also enter middle-level grades with gaps in their understanding of elementary-level mathematics.

As described in middle-level candidate expectations earlier in this chapter, middle-level mathematics teachers must know what content is taught prior to and after middle school, as well as know how to assess the extent that their students have learned what was previously taught and then address any learning gaps while also teaching the appropriate middle-level content. This knowledge and skill requires significant coursework and experiences with elementary and high school mathematics and mathematics teaching. Rational number, for example, begins with students' elementary school introduction to this number concept, a necessary foundation to accomplish the highly developed understanding of rational number expected in middle school. In a similar way, middle-level mathematics teachers must have an understanding of the concepts in algebra, geometry, statistics, and functions that are increasingly more abstract and complex. This may be accomplished in a program through the inclusion of elementary or secondary content-for-teacher courses, additional methods courses, or attention to elementary and secondary mathematics within a series of middle-level mathematics courses.

Standard P.4. Opportunities to Learn in Clinical Settings

As stated in AMLE Middle Level Teacher Preparation and Certification/Licensure guide (2015), effective middle level teacher preparation programs place a high priority on providing and requiring early and continuing middle level clinical experiences for prospective middle level teachers. The priority given these middle level clinical experiences reflects the views of practicing teachers about the essential components of professional preparation programs (Wilson, Floden, & Ferrini-Mundy, 2001).

Clinical Experiences in Middle-level Settings

Programs that support the development of well-prepared beginning middle-level teachers of mathematics include clinical experiences in middle schools that are exemplar sites, reflecting standards for mathematics and middle-level education. [Elaboration of P.4.1]

The clinical experiences of effective teacher preparation programs are guided by a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for teacher candidates to teach and to learn from their own teaching and the teaching of others.

Indicators include:

- P.4.1. Collaboratively develop and enact clinical experiences
- P.4.2. Sequence school-based experiences
- P.4.3. Experience teaching with diverse learners
- P.4.4. Recruit and support qualified mentor teachers and teacher preparation supervisors

Programs for the preparation of middle-level mathematics teachers seek placements in model middle schools. Consistent with the standards of the AMLE and expressed in the organization's landmark position paper, *This We Believe* (2010), effective middle schools address the needs of middle-level learners including an emphasis on students helping one another to be successful and developing the abilities to contribute positively to their communities and the world. Schools at which well-prepared beginning teachers serve as interns and develop their practice through clinical experiences must exemplify practices that support the needs of middle-level learners. For example, young adolescents tend to be highly curious and display a broad array of interests (which can quickly change), are eager to

learn about things they find interesting, and prefer active learning (Kellough & Kellough, 2008). Mathematics planning, teaching, and assessing practices within the school and the assigned classroom(s) must clearly value, advocate for, and understand the unique characteristics of young adolescents. Model middle schools also have organizational features such as interdisciplinary teams, learning environments, and time structures that contribute to learning and achievement while avoiding practices such as tracking that lower expectations for many learners. Furthermore, the school environment must be safe, inclusive, and supportive of the learners' needs. The involvement of families, businesses, and other members of the community must be evident and active. When such a middle school is not a possibility, middle-level candidates need opportunities to consider how such an environment can be created in their own classroom. Experiences such as videos, group discussions, and personal reflections can supplement their clinical placements.

To ensure the curriculum principles unique to middle level education are addressed, programs must ensure clinical internship or practicum placements occur not only within a mathematics classroom but also as part of an instructional team that involves teachers with licensure or certification in other disciplines. It is not necessary to have full certification in two different content areas, but it is essential for the well-prepared beginning teacher of middle school mathematics to have practical experiences teaching content that complements mathematics and is part of the diverse middle level school curriculum. In particular, the integration of information literacy skills and appropriate state-of-the-art technologies into teaching mathematics to meet the needs of all young adolescents (e.g., race, ethnicity, culture, age, appearance, ability, sexual orientation, socioeconomic status, family composition) is essential.

Standard P.5. Recruit and Retain Teacher Candidates

The recruitment and retention of high quality mathematics teachers for the middle grades offer unique challenges to educator preparation programs. Middle school mathematics certification is relatively new and in many states not required. Majors or programs specific to middle level preparation are not well known particularly among entering college students who typically sort themselves between majors or career goals tied to elementary or secondary school teaching. Often, secondary school is synonymous with grades 9-12 or high school and middle school focus and opportunities are missed.

An effective mathematics teacher preparation program attracts, nurtures, and graduates high quality teachers of mathematics who are representative of diverse communities.

Indicators include:

- P.5.1. Recruit strong candidates
- P.5.2. Address diverse community needs
- P.5.3. Provide experiences and support structures
- P.5.4. Assess recruitment and retention data

As described in Chapter 3, the Mathematics Teacher Education Partnership Research Action Cluster on recruitment has identified effective recruitment strategies for mathematics teachers. Here we revisit these strategies, as they apply to attracting high school and college students to middle-level majors, where they exist, or to programs leading to middle grades mathematics certification or licensure include:

- Offering Field experiences in middle school mathematics settings with exemplary teachers
- Providing scholarships specific to middle level programs
- Promoting the need for middle grades mathematics teachers that exceeds the need for elementary teachers as well as for middle level English/Language Art or Social Studies teachers
- Highlighting the integrated and active learning curriculum intended for middle grades learners
- Building a connection to the unique emotional and cognitive needs of middle grade learners
- Providing career counseling to elementary and secondary education as well as mathematics majors about major changes and certification options specific to middle school teaching

Majoring in X became majoring in Y. **Teaching middle school math** multiplied her opportunities.



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CHAPTER 7. ELABORATIONS OF THE STANDARDS FOR THE PREPARATION OF HIGH SCHOOL TEACHERS OF MATHEMATICS

High school mathematics teachers must have not only strong content knowledge but also strong knowledge of mathematics-specific pedagogy and much more -- including cultural knowledge about their individual students, school policies, and how to collaborate with other teachers. Only with this knowledge will mathematics teachers be able to meaningfully support the learning of all students. This chapter puts forth elaborations and examples of the standards in Chapter 2, describing the knowledge, skills, dispositions, and actions that well-prepared beginning high school mathematics teachers need to develop, followed by elaborations and examples of the standards in Chapter 3, describing what high school-level preservice programs need to do to ensure the effective preparation of their candidates. The chapter concludes with example approaches that programs might take in achieving the standards. A summary of the high school elaborations is given in Table 7.1.

Table 7.1. Elaborations of Candidate Standards for High School Teachers of Mathematics

<p>Part 1: Candidate Knowledge, Skills, and Dispositions</p> <p>Essential Understandings of Mathematics Needed to Teach High School Mathematics</p> <p><i>A well-prepared beginning teacher of high school mathematics has a solid and flexible knowledge of core mathematical concepts and procedures from the high school curriculum and the mathematical processes and practices in which their students will engage. Core mathematical concepts should include algebra as generalized arithmetic; functions in mathematics; diagrams and definitions in geometry; and statistical models and statistical inference. [Elaboration of C.1.1]</i></p> <p>Use of Tools and Technology to Teach High School Mathematics</p> <p><i>Well-prepared beginning teachers of secondary mathematics must be proficient with tools and technology designed to support mathematical reasoning and sense making, both in doing mathematics themselves and in supporting student learning of mathematics. In particular, they should develop expertise with spreadsheets, computer algebra systems, dynamic geometry software, statistical simulation and analysis software, and other mathematical action technologies, as well as other tools such as physical manipulatives. [Elaboration of C.1.6]</i></p> <p>Supporting the Opportunity to Learn by all High School Mathematics Teachers</p> <p><i>Well prepared beginning secondary mathematics teachers must understand the importance of providing all high school students with the opportunities to learn mathematics that will enable them to think analytically and creatively in preparation for workforce, college, citizenship, and life. [Elaboration of C.2.2.1]</i></p> <p>Part 2: Program Characteristics and Qualities</p> <p>Programs to Support Preparation of High School Mathematics Teachers</p> <p><i>The preparation of well-prepared beginning high school mathematics teachers requires a program</i></p>

specifically focused on preparing secondary mathematics teachers. Moreover, programs preparing candidates to teach a broader range of grades (6-12 or 7-12) should also consider the recommendations in this document for the middle level preparation.

Mathematical Content Preparation of High School Mathematics Teachers

Programs preparing high school mathematics teachers must focus on the content knowledge needed for teaching high school mathematics. This should include at least three content courses particularly addressing the needs of prospective high school mathematics teachers and should include sufficient attention to a data-driven, simulation-based modeling approach to statistics. [Elaboration of P.2.1]

Ethics and Values for Teaching

Programs preparing high school mathematics teachers should include various opportunities for beginning teachers to understand issues in order to develop their political clarity on the profession and on their advocacy role in teaching. High school teachers' role in teaching young adults during their final years of compulsory education and potential for providing the bridge to further education, employment, and citizenship heightens the need for high school teachers to attend to this advocacy role. [Elaboration of C.3.3]

Methods Courses

Programs preparing high school mathematics teachers should provide multiple opportunities to learn how to teach mathematics effectively through the equivalent of two mathematics-specific secondary-level methods courses, with at least half of the secondary level experiences targeted at the high school level. Methods courses should be offered as part of a broader program that includes other secondary-focused courses and high-school field experiences. [Elaboration of P.3.4]

Programs preparing candidates to teach a broader range of grades (6-12 or 7-12) must also attend to the recommendations in this document for the middle level preparation. Having a preparation to teach high school does not automatically prepare a candidate to teach in the middle grades.

Part 1: Elaborations of the Knowledge, Skills and Dispositions Needed by Well-prepared Beginning High School Mathematics Teachers

This section provides additional detail, commentary, and examples of the knowledge, skills, and dispositions well-prepared high school mathematics teachers should have, organized by the general standards described in Chapter 2.

Standard C.1. Knowledge of Mathematics for Teaching

Having strong subject matter knowledge is critical for all well-prepared beginning teachers of mathematics, including those at the high school level. More intensive subject matter preparation is necessary [C.1.1] given the level of the material; a more detailed outline of the essential mathematical understandings well-prepared beginning teachers of high school mathematics need is given below.

Many high school mathematics teacher candidates will have experienced success with a narrow school mathematics curriculum that did not emphasize mathematical practices and processes. Thus, it is imperative that they gain personal experience with those practices so that they will be able to fully support their students' mathematical development. Many high school mathematics teachers immediately seek to write algebraic equations, ignoring other representations and approaches that may be useful, as shown in the following problem.

Well-prepared beginning teachers of mathematics possess appropriate mathematical knowledge of and skill in mathematics needed for teaching. They engage in appropriate mathematical practices and support their students in doing the same. They can read, analyze, and discuss curriculum, assessment, and standards documents as well as students' mathematical productions.

Indicators include:

- C.1.1. Core content knowledge
- C.1.2. Mathematical practices and processes
- C.1.3. Mathematical dispositions
- C.1.4. Analyzing the mathematical content of curriculum
- C.1.5. Mathematical knowledge to analyze mathematical thinking
- C.1.6. Using mathematical tools and technology

Vignette: Meaningful Algebraic Expressions

The instructor of a methods class gave her students the following problem.

The height (in meters) of a ball t seconds after being thrown into the air is given by the the following equation:

$$h(t) = 1\frac{3}{16} + 18t - 16t^2$$

What is the maximum height the ball will reach? (adapted from FHSM, 2009)

Many students immediately thought of taking the first derivative and solving for 0, knowing that the maximum height will occur at a critical point. While acknowledging that this approach is correct, the instructor urged her students to consider other ways that they could find the maximum height. As the students worked in small groups to explore the problem further, they came up with a wide range of strategies including:

- Group 1 drew a graph of the function using a graphing utility and used it to estimate the maximum. They also noted that the graphing utility identified the maximum.
- Group 2 made a table of values to estimate the maximum. However, they further noticed that the function has zeroes at $t = \frac{19}{16}$ and $t = -\frac{1}{16}$. They reasoned that, since a parabola is symmetric, the maximum value will need to occur at the midpoint between those two values, $t = \frac{9}{16}$. So the maximum if $h(\frac{9}{16}) = \frac{100}{16}$.

- Group 3 remembered that the method of completing the square can be used to find the vertex of a parabola, which they felt should be the maximum value. But they couldn't quite remember how to do that.

In the subsequent class discussion, the instructor asked the groups present their solutions to the class, carefully explaining their reasoning, and the other groups were encouraged to analyze their approach. After Group 1 presented its solution, another student asked, "Just because it looks like the maximum on the graph, are you sure that is the exact maximum height?" The Group 1 members had to admit that they were just estimating, so it may not be exact.

Group 2's solution was met with some amazement by the other groups, since they hadn't thought of taking such a simple approach. The instructor led a brief discussion of why the graph of a parabola must be symmetric.

Finally, Group 3 showed their progress but admitted that they weren't entirely sure how completing the square worked. The instructor allowed the other groups to discuss briefly among themselves, and the class reached agreement on the algebra -- $h(t) = -16\left(t - \frac{9}{16}\right)^2 + \frac{100}{16}$. The instructor encouraged the students to discuss why this would reveal the maximum; that is, -16 times a square must be 0 or negative, so $h(t)$ must be less than or equal to $\frac{100}{16}$. She also challenged the students to represent completing the square geometrically -- "After all, doesn't it sound geometric?" She gave them algebra tiles and asked them to explore several simpler problems to better understand the method.

Finally, high school mathematics teacher candidates need to gain a positive disposition towards doing mathematics, recognizing that even in the absence of a known solution method, they have resources available that can help them make progress towards a solution. Despite challenges this may be facing in their advanced mathematics classes, they still see mathematics as an exciting and interesting endeavor [C.1.3]; without that disposition, they will have little success in convincing their students of the value and importance of mathematics.

Essential Understandings of Mathematics Needed to Teach High School Mathematics

A well-prepared beginning teacher of high school mathematics has a solid and flexible knowledge of core mathematical concepts and procedures from the high school curriculum and the mathematical processes and practices in which their students will engage. Core mathematical concepts should include algebra as generalized arithmetic; functions in mathematics; diagrams and definitions in geometry; and statistical models and statistical inference. [Elaboration of C.1.1]

The *Mathematical Education of Teachers II* (MET II) (Conference Board on the Mathematical Sciences [CBMS], 2012) suggests that well-prepared beginning teachers of high school mathematics need mathematical content that is "tailored to the work of teaching, examining connections between middle grades and high school mathematics as well as those between high school and college" (p. 54). In addition, they need to build their strength with mathematical processes and practices; as MET II suggests, they need a "full range of mathematical experience themselves: struggling with hard problems, discovering their own solutions, reasoning mathematically, modeling with mathematics, and developing mathematical habits of mind" (CBMS, 2012, p. 54) [see C.1.2]. Finally, they need to have a productive disposition towards mathematics, recognizing that mathematics is inherently a human

activity [see C.1.3]. Historically, mathematics is rooted in the development of useful tools to address problems in the real world, and recognizing that impulse provides both meaning and motivation to the study of mathematics. Acknowledging that mathematics has been created by a wide range of civilizations across history reinforces that it does not belong to a particular group.

Chapter 2 of this document suggests that *“a well-prepared beginning teacher of mathematics has a solid and flexible knowledge of core mathematical concepts and procedures they will teach and the mathematical practices in which their students will engage”* [see C.1.1]. In high school, this includes understanding algebra as generalized arithmetic, the role of functions in mathematics, the role of diagrams and definitions in geometry, and statistical models and statistical inference. As the high school mathematics curriculum continues to evolve, additional experiences in mathematical modeling and computer programming or coding may also be needed. Each of these areas is discussed below; the *NCTM CAEP Mathematics Content for Secondary Addendum to the NCTM CAEP Standards 2012* (National Council of Teachers of Mathematics [NCTM], 2012) provides more detailed descriptions of the mathematical knowledge that beginning mathematics teachers should have and also reiterates the importance of mathematical processes and practices.

Algebra as generalized arithmetic. Beginning secondary teachers understand that students in elementary grades focus on the meanings of numbers and operations on them as a foundation to develop tools and techniques in arithmetic, and that in the secondary grades they move from this focus on arithmetic to a focus on algebra as generalized arithmetic (Usiskin, 2004). Programs should give preservice teachers opportunities to focus on this view of secondary mathematics; without a purposeful focus, teachers can begin their careers relying on a naive view of algebra as “symbol pushing”—rules performed on symbols without underlying quantitative reasoning. Embedded in this understanding of algebra as generalized arithmetic is an understanding of the role and nature of variables as a part of the language of mathematics. This perspective begins to develop in the middle grades and should be addressed in courses designed specifically for a secondary mathematics teacher audience. However, this perspective can and should also be part of how mathematics is approached in courses with a broad audience, from calculus, to introduction to proofs, to linear or abstract algebra.

Functions in mathematics. Well-prepared beginning high school mathematics teachers understand “big ideas” about functions (Cooney, Beckmann, & Lloyd, 2010):

1. The concept of function is intentionally broad and flexible...
2. Functions provide a means to describe how related quantities vary together ...
3. Functions can be classified into different families of functions, each with its own unique characteristics ...
4. Functions can be combined by adding, subtracting, multiplying, dividing, and composing them...
5. Functions can be represented in multiple ways, including algebraic, graphical, verbal, and tabular representations” (pp. 7-8).

In addition to understanding the nature of functions, beginning teachers must have a facility with the foundational functions that are studied in high school mathematics: linear, quadratic, and other polynomials; rational functions; exponential, logarithmic, and trigonometric functions; and functions that are recursively defined. This understanding of functions should permeate the calculus sequence, but while the tools of calculus are required of beginning teachers, it should not be assumed that a preservice teacher who has completed the calculus sequence has had sufficient investigation of functions necessary to build an understanding of the functions of high school mathematics in enough depth to teach them well. The functions of pre-calculus level mathematics should be revisited in courses

that study high school mathematics from an advanced perspective and should be studied from a teaching perspective in methods courses.

Diagrams and definitions in geometry. Well-prepared beginning teachers understand that the role of geometry in high school has shifted from what it was in middle school, and that the objects of study in geometry in high school are “general properties of classes of figures” rather than “properties of individual figures” (Usiskin, 2004). Sinclair, Pimm and Skelin (2012) state that “working with diagrams is central to geometric thinking” (p. 9). They further identify that “geometry is about working with variance and invariance, despite appearing to be about theorems” (p. 22); “working with and on definitions is central to geometry” (p. 36); and “a written proof is the endpoint of the process of proving” (p. 48). The ability to use of dynamic software, such as GeoGebra, to investigate and understand variance and invariance in diagrams is essential.

Statistical models and statistical inference. Statistics holds a unique place in the high school mathematics curriculum. Statistics and mathematics as disciplines share a common foundation, but they are distinct areas, each with its own methods and traditions, the most notable that statistics is about inference from inductive processes, and mathematics is decidedly deductive. But, in high school, statistics is taught as part of the mathematics curriculum. It is easy, then, for beginning teachers to mistakenly believe that statistics is just another course in mathematics (like algebra, or geometry, or calculus) and undervalue the important distinctions between the two subjects.

The *Statistical Education of Teachers* (Franklin, Bargagliotti, Case, Kader, Schaefer & Spangler, 2015) describes the problem-solving experiences and habits of mind that statisticians have identified are important to their discipline. The report identifies “modern data-analytic approach to statistical thinking, a simulation-based introduction to inference using appropriate technologies, and an introduction to formal inference” (p. 8) as the appropriate introduction to statistics for high school teacher preparation. It further recommends attention to “both randomization and classical procedures for comparing two parameters based on both independent and dependent samples (small and large), the basic principles of the design and analysis of sample surveys and experiments, inference in the simple linear regression model, and tests of independence/homogeneity for categorical data” (p. 8) and a focus on statistical modeling “based on multiple regression techniques, including both categorical and numerical explanatory variables, exponential and power models (through data transformations), models for analyzing designed experiments, and logistic regression models” (p. 8). High school mathematics teachers hold the bulk of responsibility for ensuring the integrity of the statistics that is taught and that students learn in schools. Statistics, data analysis, and modeling offer the unique opportunity to address real world problems that affect students in their communities.

Peck, Gould, Miller, and Zbiek (2013) identify five “big ideas” that are central to teaching high school statistics: “Data consist of structure and variability; Distributions describe variability; Hypothesis tests answer the question ‘do I think this could have happened by chance?’; The way in which data are collected matters; [and] Evaluating an estimator involves considering bias, precision, and the sampling method” (pp. 10-11). This perspective on statistics and statistical reasoning should be present in teacher preparation programs in statistics coursework; a modeling course can address the distinctions between mathematical models and statistical models and offers an appropriate forum for discussing the distinction between mathematics and statistics in a way that prepares teachers to address it in their high school classrooms.

New emphases. While the high school mathematics curriculum has been relatively static for the past generation, new emphases are finding their way into the curriculum reflecting changes in both in how

mathematics is used and in societal emphases. For example, Guidelines for Assessment and Instruction in Mathematical Modeling Education (Consortium for Mathematics and Its Applications & Society for Industrial and Applied Mathematics, 2016) suggest the importance of mathematical modeling, “a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena” (p. 8). This is different from than the equally important work of modeling mathematics; which is addressed in many other recommendations about how mathematics is represented (Hirsch & McDuffie, 2016, p. ix-x). Investigating mathematical concepts through engineering design or a basic modeling cycle not only provides opportunities to integrate other disciplines, but also provides a means for students to improve their mathematical understandings (Usiskin, 2015). Well-prepared high school mathematics teachers should understand mathematical modeling and its potential place in the curriculum.

Calls for increasing emphasis on computer science and coding are frequently delegated to mathematics teachers, given “similarities, connections, and intersections between the fields of computer science and mathematics” (NCTM, 2016). Indeed, writing computer code can be a powerful tool for solving mathematics problems, and computer coding includes interesting mathematics related to the design and analysis of algorithms—for example, understanding why a particular program will take exponentially longer to execute as the complexity of the situation increases, suggesting that it is not a fruitful approach. However, as NCTM’s (2016) position statement on “Mathematics and Computer Science” acknowledges, well-prepared mathematics teachers must understand that computer science is not merely a subfield of mathematics but is a field in its own right that requires specialized knowledge to teach. In summary, well-prepared high school mathematics teachers need to be aware of emergent changes to high school mathematics that are likely in the coming years.

Use of Tools and Technology to Teach High School Mathematics

Well-prepared beginning teachers of secondary mathematics must be proficient with tools and technology designed to support mathematical reasoning and sense making, both in doing mathematics themselves and in supporting student learning of mathematics. In particular, they should develop expertise with spreadsheets, computer algebra systems, dynamic geometry software, statistical simulation and analysis software, and other mathematical action technologies, as well as other tools such as physical manipulatives. [Elaboration of C.1.6]

Use of technology is an expected part of society and the workforce and is an important tool for doing mathematics. Well-prepared teachers should be proficient in its use. They should be particularly prepared to use “mathematical action technologies” (cf. NCTM, 2014) useful for high school, including spreadsheets such as Excel, dynamic geometry software such as Geogebra, function graphing utilities such as Desmos and graphing calculators; computer algebra systems such as TI-Nspire and WolframAlpha; statistics simulation software such as Fathom and CoreMathTools; and other applets and technological tools that can enhance students’ conceptual understanding of mathematical concepts. These are powerful tools for doing mathematics that will be a part of the lives of the students they teach.

Use of these tools should be embedded throughout candidates’ preparation—in their methods courses, clinical experiences, and content preparation, including in courses *not* specifically designed for teachers, so that they can use them in meaningful ways. As stated in the MET II (CBMS, 2012):

Teachers should become familiar with various software programs and technology platforms, learning how to use them to analyze data, to reduce computational overhead, to build computational models of mathematical objects, and to perform mathematical experiments. The experiences should include dynamic geometry environments, computer algebra systems, and

statistical software, used both to apply what students know and as tools to help them understand new mathematical ideas—in college, and in high school. Not only can the proper use of technology make complex ideas tractable, it can also help one understand subtle mathematical concepts. At the same time, technology used in a superficial way, without connection to mathematical reasoning, can take up precious course time without advancing learning. (p. 57)

Well-prepared beginning high school mathematics teachers should be comfortable using technology to do mathematics and to effectively use technology to support meaningful mathematics learning. They should develop a disposition towards critically evaluating the appropriate use of emerging technologies and be prepared to respond to new technological tools when they become available.

While generic conveyance software such as PowerPoint and interactive whiteboards can provide valuable support for classroom instruction, such software is not inherently mathematical and does not provide the experiences essential for learning high school mathematics with technology. On the other hand, technology can provide powerful tools to enhance communication about mathematics within the classroom, such blogs and interactive platforms such as teacher.desmos.com. Technology should be viewed as an expected and vital part of the classroom, not as a replacement for effective teaching, but as an embedded tool that students use to explore mathematics.

Additionally, well-prepared beginning high school teachers should view online communications, such as the MathTwitterBlogsphere, as an essential professional resource to increase their understanding of mathematics teaching, to share their triumphs and seek insights into their conundrums, and to build professional relationships with other mathematics teachers across the nation. At the same time, they should maintain a critical ability to ascertain the value of content available on the Web and in social media and be prepared to respond to misinformation found there.

High school mathematics teachers should also be prepared use other non-electronic tools such as manipulatives in their classrooms. Contrary to common beliefs, high school students benefit from using algebra tiles, 3D models, and other physical manipulatives (cf. NCTM, 2014). Well-started beginners should have the knowledge needed to “make sound decisions about when such tools enhance teaching and learning, recognizing both the insights to be gained and possible limitations of such tools” (NCTM, 2012a, p. 3).

Standard C.2. Knowledge and Pedagogical Practices for Teaching Mathematics

Candidates seeking to become high school mathematics teachers may come into programs with an excitement about doing mathematics, but they may be less aware that many of the students they will be teaching may not share that excitement. While their inclination may be to focus on the students most like themselves, they need experiences that will help them recognize that there is much more to teaching than sharing their mathematical knowledge to those few students who are already engaged in learning mathematics. They need to develop a commitment to supporting learning by all students and to develop instructional practices that will help them achieve that commitment.

Supporting the Opportunity to Learn by all High School Mathematics Teachers

Well prepared beginning secondary mathematics teachers must understand the importance of providing all high school students with the opportunities to learn mathematics that will enable them to think analytically and creatively in preparation for workforce, college, citizenship, and life. [Elaboration of C.2.2.1]

Well-prepared beginning teachers of mathematics have a foundation of pedagogical content knowledge, effective and equitable mathematics teaching practices, and a positive and productive disposition toward teaching mathematics to support students' sense-making, understanding, and reasoning.

Indicators include:

- C.2.1. Promoting equitable teaching
- C.2.2. Planning for effective instruction
- C.2.3. Implementing effective instruction
- C.2.4. Analyzing teaching practice
- C.2.5. Enhancing teaching through collaboration with colleagues, families, and community members

Well-prepared beginning high school mathematics teachers must advocate for and assure that each high school student is given the opportunity to learn mathematics in meaningful ways. All too often, whether intentional or unintentional, secondary mathematics teachers become gatekeepers to students' opportunities to engage in coursework that will prepare them for a myriad of job opportunities and life events. A teacher, who has high expectations for his or her students, poses and maintains the level of high cognitive demand tasks, questions students in meaningful ways, and facilitates discourse that leads to students developing a relational understanding of mathematics (Skemp, 1976), is likely creating optimal opportunities for students to learn. We also want students to understand how a concept relates to forms of mathematics, to themselves, and to the broader world. On the other hand, if a teacher creates an environment where students are mainly engaged in memorizing rules and mimicking what the teacher does, then the students are not going to develop meaningful mathematical knowledge and skills needed for their future success, not just in continuing education or the workforce, but in understanding how mathematics can contribute solutions to broader issues facing society, such as poverty, cancer, or access to adequate housing.

Thus, well-prepared beginning high school teachers need to understand that their beliefs about what it means to learn mathematics and their stereotypes and beliefs related to students' cultural backgrounds, ability levels, and other defining characteristics impact how they interact with students, which determines how students will engage in mathematics and how much students will achieve. Teachers' beliefs and school policies related to tracking have caused many students to be placed in courses where they do not receive instruction that enables them to reason and make sense of mathematics; instead those students become less confident in themselves as learners and doers of mathematics and might not see the point of doing mathematics if it does not relate to their current or future lives. A number of scholars have shown that tracking leads to more unfavorable outcomes than positive ones (Oakes, 2008; Horn, 2006; Stiff & Johnson, 2011). Moreover, tracking or placement of students in particular courses is

often based on demographic factors more than students' actual knowledge and abilities (Oakes, 2008; Stiff & Johnson, 2011). When mathematics teachers hold the powerful belief that all students, regardless of race, ethnicity, gender, socioeconomic status, language use, sexual orientation, immigrant status, or past performance in coursework, can and will succeed in learning important mathematics, they give their students an advantage in learning mathematics that will prepare them for their futures.

Following is a vignette that could be used with teacher candidates to discuss issues related to opportunities to learn. Mathematics teacher educators might ask teacher candidates the following questions to orchestrate a discussion focused on the vignette:

1. What might be some of the reasons for the teacher taking a hands-off approach to some of the students on this particular day?
2. Why might some of the classrooms at the school have such small numbers?
3. What would have been a better approach if the teacher were trying to help the students to develop more autonomy?
4. What are some strategies that could have been used to ensure that each one of the students was moving forward in his/her learning?

Vignette: Providing Opportunities to Learn

This excerpt describes events that took place in a pre-algebra classroom.

There were 4 students in the class today: Antonio, Vanessa, Alicia, and David. Three of the students are Black (Antonio, Vanessa, and Alicia) and one is White (David). As the class started, Vanessa asked me for help with her assignment. I assumed that the class would go as it had yesterday, with me working with Vanessa (since she was ahead of the others in the book) and Ms. Smith, the regular classroom teacher, working with the rest of the class. However, this is not what happened. I worked with Vanessa. But Ms. Smith did not work with the other students. David asked for and was given permission to go to the nurse's office. He was gone for most of the period. Antonio appeared to try a few times to get started on his assignment, but he seemed unsure how to begin. He asked Vanessa at least twice to come over to his desk and help him. But Ms. Smith refused to allow Vanessa to work with him. Antonio did not ask Ms. Smith for help. Nor did she offer him any assistance. He appeared to get little, if any, work done during this class period. Similarly, Alicia appeared to be trying to answer the questions in the book, but, given what we have seen in the past, I have to wonder how much she understood of what she was doing. Like Antonio, she did not ask Ms. Smith for help and no assistance was offered. Ms. Smith sat at her desk. She did not speak to Antonio or Alicia except to tell Antonio that he could not get help from Vanessa. At some point Ms. Smith began to make the model staircase that the students would be asked to construct in the next section of the book. When David returned to the classroom, he asked if he could also build a staircase. He worked on the model for the rest of the period. Thus, the class period ended with essentially no instructional interaction between teacher and students. After class, Ms. Smith indicated that she was intentionally taking a hands-off approach in which she would not give help until the students took the initiative to ask for it. This action (or, more accurately, inaction) appeared to be targeted at Antonio. Ms. Smith stated that he makes no effort and only disrupts the class. This appeared to be the justification for her hands-off approach.

We share this example because it was, in fact, representative of a larger pattern. What happened in this class period was not unique in our observations of the teachers' classes. The particular conditions of this class, specifically the small class size, highlight a phenomenon that could be seen elsewhere. We characterize this pattern of action and inaction as "allowing students to fail." In the example of Ms. Smith's class, her hands-off approach meant that Antonio and Alicia, the only two students in the class in need of help, went for an entire 50-minute class period without any instructional interaction with the teacher. Yet, this was but a starker example of events that took place in other teachers' classrooms as well. There were other cases in which students sat for the entire class period, essentially ignored by their teacher. These students were allowed to not work as long as they did not disrupt the class. Students who did not "take responsibility for their own learning" were subject to this hands-off approach. (Rousseau & Tate, 2003)

The preceding vignette may cause some teacher candidates to gasp in disbelief and others to say that they have witnessed such situations in school placements or as students in high school. The discussion of the vignette or similar experiences should cause them to think about what it means for students to have the opportunity to learn, and the role of teacher as an advocate for students not simply a disseminator of information. Teachers need to engineer classrooms that will provide students appropriate content exposure, content coverage, content emphasis, and quality instructional delivery (Tate, 2005).

Within their programs, secondary teacher candidates need to see students from different socioeconomic status, race/ethnicity, gender, sexual orientation, immigrant status, religious backgrounds, abilities, linguistic, and other groups reasoning and making sense of mathematics. Here is a vignette that can be used as a catalyst in a methods course around developing positive student mathematics identities.

Vignette: Mathematical Identity

I am Michael Davis, an African American junior in a geometry class. I am motivated to do well in this class because I want to go to college, and if I make good grades I might get a scholarship. I actually like this class because I am given the opportunity to solve problems in groups with my peers. I like the discussions that we have, especially debates when we do not agree on a solution. I like activities that allow me to discover important connections: between different topics of mathematics, between my life and mathematics, and between mathematics and what is going on in social media and the world. I also like making conjectures and testing them to find out if they are true.

Furthermore, teacher candidates need the opportunity within their programs to facilitate the mathematics learning of students from the aforementioned groups. Also, when possible it is important for secondary mathematics teacher candidates to collaborate with special needs teachers and teacher candidates who are focusing on special needs students. They need to learn how to read Individual Education Plans (IEPs) for students and work with the special needs teacher to understand how to differentiate instruction, use multiple entry-level tasks, and other strategies to ensure that mathematics learning is occurring in a meaningful way for each and every student.

Secondary teacher candidates also need opportunities to interact with emergent bilingual students and to understand that it is not enough to just translate words for students; they need to create environments and learning situations where the students can understand and own the mathematics for themselves. For example, students may be allowed to use algorithms from their home countries or to show their work in ways that are not standard practice in the US (Gutiérrez, 2015; Civil & Planas, 2010; Civil & Menéndez, 2011).

Secondary teacher candidates also need to understand how mathematics structures and vocabulary can be challenging for emergent multilingual students in multiple ways, such as words having multiple meanings, different meanings in different contexts, and different pronunciations when used as a noun or adjective. Thus, beginning high school teachers need to be aware of the ways in which language influences learning, including recognizing that all mathematics students are language learners (e.g., the word "function" does not mean the same thing to a monolingual English speaker when stated in the context of a broken washing machine at home versus in a math classroom). The vignette below highlights a secondary teacher candidate's revelations, as she learned how to meet the needs of her emerging bilingual students.

Vignette: A Student Teacher's Revelation Related to Emergent Bilingual Students

On the first day of school, my cooperating teacher was informed that there would be 3 Guatemalan students who did not speak English joining her 7th period algebra course. I was immediately excited and couldn't wait to meet them. I have studied Spanish for many years and now have a bachelor's degree in the language, but I have not had many opportunities to use my knowledge in the workplace. I knew that speaking Spanish would be useful as a teacher in the public school system, but I wasn't expecting it to come in handy so soon. Because of their language needs, I have been working with this particular group of students every day since day 1. This experience has given me a deeper look into the challenges that non-English speaking students face on a daily basis at schools where teachers have not been prepared to facilitate their learning well and what is necessary to help them achieve success.

During observation weeks, I spent my time in 7th period sitting with the Guatemalan students and providing explanations in Spanish to help them understand what was going on in class. In the first days of class, which consisted of pre-algebra review, simple translation of the materials and interpretation of the teacher's lessons were enough to keep the group of students on the same page as the rest of the class. However, as the ideas discussed in class quickly became more complex, I realized that translations did not suffice. They understood the words that I was saying, but not the math. It was then that I realized that what the students needed was instruction that they could understand, just like any other student. I began to shift my focus from translation and interpretation to teaching math. This was a much more difficult job and required much more thought and preparation. The same materials didn't necessarily work for both languages and both cultures, and I needed to differentiate my instruction. Thankfully, because I was still in observation weeks, I was able to provide these materials and provide one-on-one assistance.

As I continued to spend all period right next to them, I couldn't help but wonder what the 3 students did in their other classes where they did not have any Spanish-speaking people to help them. I decided to follow them through their schedule for a day and find out what they experienced. To make a long story short, they did nothing all day. They were ignored and not even given assignments in all but one class (Spanish class). I don't know what the teachers should have done with students they can't

communicate with, but I feel like doing nothing is the wrong answer.

When I began teaching 7th period myself, however, I realized that providing all of the extra help to the Guatemalan students was very difficult when there were 25 other students in the classroom to worry about (even though I spoke Spanish) I felt that my ability to help the emergent bilingual students to the degree that I would like to diminished, and I started to see the real struggle of how to help the struggling students without completely ignoring the other students.

I decided that the way one must do this is not to separate the struggling students from the rest of the group. I began designing lessons that engaged both groups of students and called on multiple levels of knowledge to allow multiple levels of students to interact with each other. Additional assistance may be provided, but, in general, we must make sure that our lessons meet the needs of each and every student.

One of the things that really stood out to me the more I worked with the Guatemalan students was their strong desire to participate and be a part of the class. I had seen just how little they were able to engage with anyone in their other classes, and I think they saw math class as one of their few opportunities to interact with other students and teachers. I wanted to use this desire not only to motivate them to understand and ask questions, but also to motivate other students to participate.

They picked up on the numbers in English very quickly and within a few weeks were calling out answers. This is in part due to the effort that their English teacher made to reinforce words from other courses in her class. They stopped saying numbers to me in Spanish and stated them only in English even when we were talking privately. I helped them to start a mathematical dictionary in their notebooks with words like equalities and inequalities; and it took them no time to start explaining their processes using these words. I was so impressed by their fearlessness in class. They did not speak English well, but they never hesitated to give solutions in front of the class. I believe that the other students in the class took note of the efforts they were making in math class. I feel that this helped encourage other students to speak out and share their own understanding.

One student in particular had a really hard time keeping up with the other two. I learned from him and the Spanish teacher that this student had not been to school in several years before coming to the US. He was clearly behind in math but also was not very strong in reading, writing, or the Spanish language, as his first language was Xinca, a language spoken by indigenous people in Guatemala. It became clear to me that just being in class would not be enough for him. After a test that he did poorly on, I spoke with him and told him that if he wanted to succeed in the class, he needed to start seeing me before school. He agreed, and the next morning he was there.

We worked together one-on-one for 20 or 30 minutes each morning and he eventually caught on to what we were doing in class. It was still a challenge knowing when to keep working on a topic and when to move on with the class, but we tried to use the extra time as wisely as possible. After a few weeks, he started coming to me less and less, and became less dependent on me during class. He felt more comfortable asking questions to his peers and more confident on quizzes and tests. Eventually, he stopped coming in the mornings, and I felt confident that he would be able to follow right along with everyone in the classroom.

This experience working with this class and these students has taught me a lot about how to effectively support students who have unique language needs. I realized that the students had come into our mathematics classroom eager to participate and wanting to be part of the classroom. I simply needed

to find the proper ways to support them to make sense of the US conventions we often use and to clarify some of the language when there were not easily identifiable cognates. I learned that rather than lumping all of the Guatemalan students into one group that I needed to probe further for what previous experiences they had in mathematics, how each one learned, what resources and expertise they already had before entering my classroom and what they needed from me in order to succeed. I have learned that it is important not to lower the expectations of students based on their language needs and that they can achieve the same learning objectives in different ways. I have also learned to recognize when a student truly needs extra time outside of class to be able to catch up with their peers. It is important to always keep in mind that providing the same learning experience for all students does not necessarily mean that the environment is equitable. All students should have the same opportunity to learn and succeed. Providing this may require some adjustments or supplemental instruction, and incorporating these methods into a curriculum that takes advantage of and celebrates the diversity of learners will benefit not only the students that you are targeting but students from different backgrounds and the resources they bring to the classroom.

This story illustrates that it is important that teacher candidates not only have experiences to work with a diversity of learners but also to reflect on and process those experiences in a way that enhances their understanding of how they can build on the cultural and linguistic resources that students bring to the classroom to support the learning of all students. This vignette may be used with a methods course to help teacher candidates think about what they would do if faced with a similar situation.

Standard C.3. Knowledge of Students as Learners of Mathematics

The discussion in the previous section is built upon the need for well-prepared beginning teachers of mathematics to understand their students' mathematical knowledge, skills, and dispositions. Well-prepared beginning teachers of mathematics need to recognize that their students may not think about mathematics the same way they do. They need to see their role as building on the thinking of their students rather than "transmitting" their mathematical knowledge to their students [C.3.1]. For example, when given a quadratic equation to solve, students may use a "guess-and-check" strategy. Rather than counting this strategy as wrong since it is not deductive in nature, well-prepared beginners will instead consider how that solution could become part of the discourse around how to solve quadratic equations; while acknowledge the thinking behind that approach, also using its limitations in situations where solutions are not integral to motivate the need for other approaches. Note that their students are also naturally engaged in the doing of mathematics, developing mathematical processes and practices [C.3.2].

Well-prepared beginning teachers of mathematics have foundational understandings of students' mathematical knowledge, skills, and dispositions. They also know how these understandings can contribute to effective teaching and are committed to expand and deepen their knowledge of students' as learners of mathematics.

Indicators include:

- C.3.1. Students' thinking about mathematics content
- C.3.2. Students' engagement in mathematical practices
- C.3.3. Students' mathematical dispositions

By the time they are in high school, many students may have come to the conclusion that they are not good at mathematics and certainly do not like studying it, and well-prepared beginning teachers need to

be cognizant of promoting their students' confidence in doing mathematics, as well as their appreciation for its usefulness [C.3.3].

Standard C.4. Social Contexts of Mathematics Teaching and Learning

The elaboration of Standard C.2 for high school stated that well-prepared beginning high school teachers need to understand that their beliefs about what it means to learn mathematics and their stereotypes and beliefs related to students' cultural backgrounds, ability levels, and other defining characteristics impact how they interact with students, which determines how students will engage in mathematics and how much students will achieve.

Realizing that the social, historical, and institutional contexts of mathematics impact teaching and learning, well-prepared beginning teachers are knowledgeable about and committed to their critical role as advocates for every mathematics student.

Indicators include:

- C.4.1. Access and advancement
- C.4.2. Mathematical identities
- C.4.3. Students' mathematical strengths
- C.4.4. Power and privilege in the history of mathematics education
- C.4.5. Ethical practice for advocacy

Part 2: Elaborations of the Characteristics and Qualities Needed by Effective Programs Preparing High School Mathematics Teachers

This section provides additional detail, commentary, and examples what preservice programs need to do in order to effectively prepare their students to teach high school mathematics. We begin with general discussion about the nature of high school mathematics teacher preparation programs. Further elaboration is provided organized by the Standards in Chapter 3.

Programs to Support Preparation of High School Mathematics Teachers

The preparation of well-prepared beginning high school mathematics teachers requires a program specifically focused on preparing secondary mathematics teachers. Moreover, programs preparing candidates to teach a broader range of grades (6-12 or 7-12) should also consider the recommendations in this document for the middle level preparation.

An effective program that provides coursework and experiences focused on the teaching of secondary mathematics is essential to the development of a well-prepared beginning high school mathematics teacher. All too often, the prevailing attitude is that secondary mathematics teachers only need a degree in mathematics, and they will be prepared to teach. While having a strong content background is important, this is not sufficient (CBMS, 2012). Others may emphasize the need for general courses on pedagogy or education with related field experiences. While there is value in such a background, it alone is also not sufficient. A well-prepared beginning secondary mathematics teacher needs coursework and experiences that build their understanding of both the content of, and teaching methods particular to, secondary mathematics, given the social context in which mathematics teaching occurs.

An analysis of various degrees at a major state university revealed some startling results about the preparation of teachers (Strutchens, 2012). Most degree programs required a significant number of hours specific to the major. For example, a degree in electrical engineering required 49 credit hours in coursework specific to electrical engineering, 31 credit hours of supporting content courses (mathematics and physics), and 15 credit hours in general principles of engineering. In contrast, a degree in secondary mathematics education at this university could include as few as 22 credit hours specific to mathematics education (which also includes a student teaching experience), with 42 credit hours in mathematics and 15 credit hours in education more generally. Clearly, the balance of coursework in mathematics teacher preparation is out of alignment with other professional preparation programs. Students in secondary mathematics education need and deserve coursework that specifically prepares them for success in their field of study, including mathematics-specific methods courses and mathematics content courses specific to teaching, and meaningful clinical experiences in secondary mathematics classrooms overseen by supervisors who have expertise in secondary mathematics across the grades they will be certified to teach; these are described in more depth in the following sections. Chapter 3 of this document indicates, *An effective mathematics teacher preparation program provides opportunities for prospective teachers to learn, with understanding and depth, the school mathematics content they will teach*. The example pathways at the end of this chapter provide an illustration of what that might look like in a secondary teacher preparation program.

Many high school teacher certification or licensure programs may include some or all of the middle grades; for example, a license may be valid for grades 7-12, or 6-12, or even 5-12. Such programs must also attend to the recommendations in this document for the middle level preparation. Having a preparation to teach high school does not automatically prepare a candidate to teach in the middle grades. Preparation for teaching mathematics in grades 6 through 8 as addressed in Chapter 6 requires that well-prepared beginners have knowledge and skills about learners at this stage of their cognitive and affective development, about the intended curriculum at the middle grades that calls for integrative approaches, and experience in teaching within interdisciplinary teams. Certainly, preparation for grades 9-12 mathematics teaching provides many of the attributes needed to be effective with middle grades learners, but the significant differences among learners at these different levels as well as the differences in how middle and high school curriculum and schools are organized require that programs that prepare future teachers for broader grade ranges address standards in both this chapter and the previous one. The following standards from Chapter 6 are of particular importance for board programs: Standards 6.1 (proportional reasoning and numbers), 6.3, 6.4, 6.5, 6.7, and clinical experiences in the middle grades 6.10 and 6.11. As stated in MET II: “[O]ne theme of this chapter is that the mathematical topics in courses for prospective high school teachers and in professional development for practicing teachers should be tailored to the work of teaching, examining connections between middle grades and high school mathematics as well as those between high school and college” (CBMS, 2012, p. 54).

This recommendation may present challenges for programs with small numbers of secondary mathematics education teacher candidates. They may not have adequate enrollments to be able to offer coursework specific to mathematics education or be able to support clinical supervisors with mathematics-specific expertise to support candidates at the high school level, let alone a broader range of grades. The MAA CUPM report on high school mathematics (Tucker, Burroughs, & Hodge, 2015) provides a number of options such programs might consider, including “regional consortia, distance learning, or co-convening courses with in-service teachers seeking graduate credit or professional development.” Alternatively, they might focus on providing an outstanding preparation in mathematics, with the intent that students intending to pursue a career in teaching mathematics enroll in a graduate program preparing them to teach. Just as not every college or university has the capacity and resources

needed to offer a degree in electrical engineering or other specialized majors, so it must be acknowledged that not every college or university has the capacity and resources needed to offer a degree in secondary mathematics education. Institutions that cannot offer the necessary experiences should strive to develop the necessary partnerships to ensure a quality preparation or else leave the work of preparing secondary mathematics teachers to others.

Standard P.1. Establish Partnerships

While partnerships are important at all levels of mathematics teacher preparation, they are particularly important in ensuring the effective preparation of high school mathematics teachers. Given that many high school mathematics teachers candidates will take many more hours in mathematics content courses than they do in coursework particularly focused on mathematics teaching, it is essential that mathematics departments

recognize and attend to the particular needs of secondary mathematics majors. Having close working relationships between mathematicians and mathematics educators will help to ensure that both courses specifically focusing on mathematical knowledge for teaching and those taken by a wider range of majors include relevant attention to the needs of secondary mathematics majors. In addition, mentor teachers overseeing clinical experiences need to be included in discussions around secondary mathematics teacher preparation to ensure alignment between the practices advocated within the preparation program and the instructional practices candidates both observe and are supported in implementing. Having close relationships among those responsible for the program and the mentor teachers beyond a sole focus on providing clinical experiences enhances the probability this will happen. The following vignette illustrates this point.

Effective programs for preparing mathematics teachers have significant input and participation from all appropriate stakeholders.

Indicators include:

- P.1.1. Engage all partners productively
- P.1.2. Provide institutional support

Vignette: The Importance of Coherent and Consistent Program Components

The prospective mathematics teachers enrolled in State University's one-year post-graduate secondary credential program love the two mathematics methods courses they take from Robert. He makes the course engaging yet relevant, and the students appreciate the innovative and empowering ways they are learning to think about mathematics, mathematics teaching, and students. State U. offers a fifth-year teacher credential program, whereby students are enrolled in cohorts of 30 students, with each cohort led by a leader who is responsible for placing students into their student teaching assignments. The students in each cohort together take a set of non-subject specific courses, including social foundations, educational psychology, reading in the content area, teaching content to ELLs, and classroom management, and once a week the secondary students from the different cohorts meet to take their content-specific methods courses. Consequently, Robert teaches the mathematics methods courses to all of the secondary credential students. The students concurrently student teach, so Robert provides opportunities in his methods class for students to reflect on opportunities and challenges experienced by student teachers.

Over the years, one of Robert's major frustrations has been that student teachers are generally not observing the kind of instruction from their guide teachers that he has been teaching about in his methods class. Robert has found that his class's message is often trumped by the classroom practice the candidates observe, and when Robert raised the issue with others in his department, the response

was that student teaching placements have always been made by the cohort leaders, who must consider such factors as the school's location, the principal's sense of the needs of his/her school, and convenience for student teaching supervisors.

Lately, however, the situation has dramatically improved as a result of two colleagues, Lauren and Diane, being awarded a Noyce Master Teaching Fellows grant. Lauren and Diane recruited 40 exceptional secondary school mathematics and science teachers who began a five-year professional development project focused on improving classroom instruction. Furthermore, because one of the goals of the Noyce Master Teaching Fellow Project was leadership, Lauren and Diane supported the Noyce teachers in emerging into leaders, including how to more effectively serve as guide teachers to student teachers. After two years of the Noyce project, Lauren and Diane lobbied for and were permitted to create and lead a new mathematics/science cohort, whereby 30 of the secondary mathematics and science credential students could apply for and be accepted into this STEM-focused cohort. As part of this new cohort, Lauren and Diane assumed the responsibility for assigning student teachers, and their highest priority was that the guide teachers be selected on the basis of their excellent and innovative instruction. Furthermore, they arranged for many of the Noyce Master Teacher Fellows to serve as guide teachers. For the first time, Robert has found that most of his mathematics methods students are now seeing the kind of instruction in their assignments that he is encouraging in his mathematics methods class.

This vignette highlights the complexity of program effectiveness, which can be attained only through the coordination and shared goals among all those charged with teaching or supporting the candidates. Even the greatest mathematics methods class will have minimal effect on the students if the methods class stands alone, disconnected from other program components. Candidates must experience a coherent message among the many moving parts across the university and the local school partners. This example also highlights that commitment to a credential program is a necessary, but often insufficient, condition for program excellence, and in this case the impetus for program change was initiated by the professional activity of the faculty. In particular, funding of the Noyce grant set into motion a series of changes that led to an improved credential program. Without the symbiotic relationship among the mathematics educators' work inside and outside the credential program, the program improvement would not have occurred.

Standard P.2. Opportunities to Learn Mathematics

In this section, we provide commentary on how high school mathematics teacher preparation can support their candidates' opportunities to learn to mathematics, particularly the specific content preparation they need.

Mathematical Content Preparation of High School Mathematics Teachers

Programs preparing high school mathematics teachers must focus on the content knowledge needed for teaching high school mathematics. This should include at least three content courses particularly addressing the needs of prospective high school mathematics teachers and should include sufficient attention to a data-driven, simulation-based modeling approach to statistics.

[Elaboration of P.2.1]

An effective mathematics teacher preparation program provides beginning teachers with opportunities to learn mathematics that are purposefully focused on essential big ideas across content and processes that foster a coherent understanding of mathematics for teaching.

Indicators include:

- P.2.1. Attend to mathematics content for teaching
- P.2.2. Build mathematical practices and processes
- P.2.3. Provide sustained experiences

Our standards echo the recommendations of chapter 6 of the MET II (CBMS, 2012) and the *Committee on the Undergraduate Program in Mathematics Guide to Majors in the Mathematical Sciences* (CUPM Guide) (Tucker, Burroughs, & Hodge, 2015). Additionally, these standards incorporate recommendations in *The Statistical Education of Teachers* (Franklin et al, 2015). We reiterate the CUPM Guide's point of view that a "traditional liberal arts major in mathematics is neither necessary nor sufficient preparation for teaching high school mathematics" (p. 1). Full descriptions of programs that lead to a mathematics major that *do* provide the necessary preparation for teaching are provided in the CUPM Guide. A mathematics major that is tailored for prospective teachers should contain lower division content in calculus, linear algebra, mathematical proof, and data-based statistics with a focus on statistical inference. Upper division coursework in mathematics can be organized around a variety of topics; our main recommendation is that the equivalent of 9 semester-hours of coursework focus on "high school mathematics from an advanced standpoint" (CBMS, 2012, p. 55). The CUPM Guide provides examples of organizing this coursework around geometry, algebra, analysis, modeling, number theory, discrete mathematics, or the history of mathematics; we recommend that programs include some attention to all of these content areas, within courses designed specifically for teachers where possible. In addition to a focus on mathematics content, the mathematical practices of reasoning and sense-making should form the foundation of mathematics coursework. Programs should attend to the increasing importance of statistics and ensure that prospective teachers engage with statistical ideas through three courses that follow the design principles articulated in *The Statistical Education of Teachers* (Franklin et al, 2015).

As a philosophy underlying the mathematical education of secondary teachers, all mathematics courses that include prospective teachers (not just those specifically targeting prospective teachers) should provide examples and opportunities for students to examine how related mathematical ideas exist within the secondary curriculum. This can be done as naturally and automatically as examples are provided for prospective engineers, or nurses, or business executives within most mathematics courses. In recognition that fitting appropriate experiences into existing programs can be challenging, many researchers are focusing on providing such examples. For instance, the MTE-Partnership's MODULE(S)² Research Action Cluster is developing modules related to key areas of mathematics content for teachers (e.g., geometry, statistics, and modeling). These unit-long modules can either be bundled into a domain-

specific course (e.g., 3 modules on transformational geometry) or into a “capstone” course including modules from across domains. Alternatively, they can be inserted into an existing mathematics course that may be taken by students from a variety of majors, such as including one of the geometry modules into a Euclidean geometry course. Some instructors have even included a module into a mathematics methods course. Collaborative efforts like this are helping to create resources to address the ongoing challenges of creating programs with high standards for teacher preparation, such as we describe here.

An ongoing challenge is the different ways in which states require teacher certification to take place. Some states are coursework-based, others are content-exam-based (such as a minimum score on the Praxis II), and some are both. Some programs prepare teachers within a 120-credit 4-year degree that includes mathematics content, education coursework, and student teaching; others are fifth-year post-baccalaureate programs; and still others include a fifth year master’s degree including initial teacher preparation. These differences preclude our recommendations from being one-size-fits-all. In the examples at the end of this chapter, we provide sample high school teacher preparation programs from a variety of perspectives, fully recognizing that these illustrations do not completely capture the variation of the possibilities that exist. A particular challenge for fifth-year programs is that prospective teachers may come to the program with an undergraduate mathematics degree that was designed for the purposes of preparing a student for graduate school in mathematics; providing them with both additional mathematics content from the perspective of content knowledge for teaching and education coursework may prove challenging. However, that background is essential for well-prepared beginning high school mathematics teachers.

Finally, we reiterate our previous point that high school teacher certification or licensure programs that include some or all of the middle grades need to attend to the content needed for middle grades mathematics as described in Chapter 6.

Standard P.3. Provide Opportunities to Learn to Teach Mathematics

In this section, we provide commentary on how high school mathematics teacher preparation can support their candidates' opportunities to learn to effectively teach mathematics -- Ethics and Values for Teaching and Methods Courses.

Ethics and Values for Teaching

Programs preparing high school mathematics teachers should include various opportunities for beginning teachers to understand issues in order to develop their political clarity on the profession and on their advocacy role in teaching. High school teachers' role in teaching young adults during their final years of compulsory education and potential for providing the bridge to further education, employment, and citizenship heightens the need for high school teachers to attend to this advocacy role. [Elaboration of C.3.3]

Chapter 2 of this document indicates, Well-prepared beginning teachers of mathematics are knowledgeable about and accountable for enacting ethical practices that allow them to advocate for themselves and to challenge the status quo on behalf of their students (C.4.6). In high school, this includes understanding the possible influences of the curriculum and school policies on student learning and students' identities and being willing to stand up for their students when their best interests are being violated. Chapter 3 indicates, Well prepared beginning mathematics secondary teachers need to understand the importance of providing all high school students with the opportunity to learn mathematics. Rather than lumping all students into one category, having political clarity means ensuring that the very students who have not been served well by the school system become the focus and the means by which we judge the efficacy of what we are doing in mathematics teaching.

Mathematics methods courses must also address the social, historical, political and institutional contexts that affect mathematics teaching and learning and provide practice-based experiences to develop core practices and pedagogical content knowledge that honors mathematics, students' mathematical thinking, and cultural/community-based funds of knowledge and experiences. When considering mathematics methods courses, instructors must be aware of issues particular to high school students. For example, although the practice of sorting students into or away from high level mathematics courses begins before high school, these tracks are often "set in stone" by high school, thereby precluding many students from accessing the most rigorous mathematics courses. Furthermore, although deficit language pervades all levels of schooling, the culture of some high schools is such that deficit language is ingrained in the fabric of the mathematics department. Moreover, many teachers may feel that by the age of high school, those students who are "good" at mathematics have already been defined and little can be changed in students' dispositions towards the discipline or their motivations to learn. For example, it is not uncommon for teachers to gather in the lunchroom or math office to complain about how little skills students have today, versus previously, or to speak about groups of students as if they are a homogenous group, ignoring their individual differences and the

An effective mathematics teacher preparation program provides candidates with multiple opportunities to learn how to teach through mathematics-specific methods courses that integrate mathematics, knowledge of students as learners, the social contexts of mathematics teaching and learning, and practices for teaching mathematics.

Indicators include:

- P.3.1. Address deep and meaningful mathematics content knowledge
- P.3.2. Provide candidates with strong foundations of knowledge about students as mathematics learners
- P.3.3. Address the social contexts of teaching and learning
- P.3.4. Provide practice-based experiences
- P.3.5. Provide effective mathematics methods instructors

unique strengths and experiences they bring to classrooms. Methods classes should offer opportunities for prospective teachers to learn how to respond to and professionally challenge colleagues who may hold deficit perspectives on students.

For example, rather than simply learning about the latest reform initiatives in mathematics education, prospective teachers can benefit from opportunities to see that professionals do not always blindly follow what their district or state officials recommend. Instead, prospective teachers can be exposed to both the need and specific strategies for creative insubordination, the bending of rules to adhere to higher ethical standards (Gutiérrez, 2013, 2016), when working with high school teaching colleagues. One way in which secondary mathematics teacher education programs can accomplish this work is through offering opportunities for prospective teachers to analyze the context of mathematics teaching and learning and prepare for the political realities of the profession. That is, just as prospective teachers need time to rehearse for teaching through such activities as peer teaching, or extensive lesson planning, they need opportunities to prepare for the kinds of controversial discussions and negotiations they will face as new teachers in a context of high stakes education and in a country that has consistently failed to support Black, Latin@, American Indian, and other historically marginalized communities. If a prospective teacher does not feel a school policy or curricular initiative is fair, they will need to practice challenging that policy, including knowing how to do such things as effectively “press for explanation,” “turn a rational issue into a moral one,” or “seek allies.” They should understand that a number of factors will help them be more strategic and effective. According to Gutiérrez (2016), “Choosing an appropriate strategy requires we first recognize the kind of issue at stake (i.e., What power dynamics are operating? How does this issue relate to student learning and social justice?) and then consider the speaker(s), our relationship with them, and the context in which we find ourselves.” Given the growing knowledge base of both prospective and practicing teachers using creative insubordination, teacher education programs can draw on cases of effective teachers employing this practice.

The following vignette provides an example of the kinds of dilemmas and reflective exercises that secondary mathematics teacher education programs may provide as a means for beginning teachers to consider the ethical decisions they may face in their future teaching.

Vignette: Helping Candidates Consider Ethical Decisions They May Face

Marcia is attending her mathematics department meeting. She likes her colleagues and generally agrees with their approaches to teaching. However, today, she learns that the district decided to change their textbooks to a series that is text heavy and requires significantly more reading for students to understand the examples and begin the homework. She recognizes the need for occasional changes in texts to keep up with current reforms, but she is worried that the emergent bilingual students in her school may now require more of her attention because they may be less likely to use the text as a resource or they may be less likely to persevere in text-heavy word problems in homework sets. She asks herself: Will they get confused with all of the extraneous words or unfamiliar contexts? Many of the students she knows who speak more than one language have been transitioned out of bilingual education classes and may not appear to need any special attention. She would like to raise this issue at her meeting. But, she is only in her 2nd year of teaching and wonders if she should just stay quiet on policy issues until she has earned tenure. She hopes someone else in her department will speak up on behalf of the emergent bilingual students in their school. But, after some time in the meeting, she realizes that none of her colleagues recognize the

potential hurdle these new texts may pose for students. A secondary mathematics teacher education program could ask beginning teachers to take some time to reflect on how they might handle the situation if they were in Marcia's shoes.

1) Suppose Marcia decides to speak up. How might she frame her concerns so that it does not sound like she is simply complaining and has no solutions? What kind of language could she use in this public meeting that will promote dialogue and action, rather than self-defense or resistance by her colleagues? Are there particular strategies for creative insubordination that she can employ? What kinds of examples or information might elicit understanding and empathy from her colleagues or administrators rather than deficit perspectives towards students? How might she speak to individuals who do not share her expertise or commitment to emergent bilinguals? If they decide as a department to go along with the district's choice, could Marcia convince the department to structure ways to support students with the new texts? What might that look like? Is there a way to retain the old texts as resources for students? If Marcia is unsatisfied with the outcomes of the meeting, are there things she can do alone with her emergent bilingual students that would not require knowledge by or consensus with other teachers?

2) Suppose Marcia decided not to speak up at the meeting, but wants to follow up later on. How might she decide where to direct her energies? Would raising the issue with the department chair be the right place to start? Or, should she begin with a sympathetic colleague who has more teaching experience or more years in the school? What role could data or research articles play in her attempts to convince her colleagues about a particular action? Is there a role that students and/or parents might play in helping her advocate for emergent bilingual students and others who may struggle with the new textbook? Are there ways for her to lean on allies (e.g., non-mathematics teachers in the building) who may have more seniority? What might a collective effort look like?

Such reflective exercises might require prospective teachers to develop an essay that provides "professional advice" to Marcia; produce a script; or undergo a role play where they rehearse what it might look like for Marcia to stand up to the district and enlist the help of others in her mathematics department to advocate for the emergent bilinguals in the school. This practice of rehearsing through the activity of "In My Shoes" is being used in mathematics teacher education programs in various universities across the US (see for example, Gutiérrez & Gregson, 2013; Gutiérrez, Gerardo, & Vargas, in press).

Finally, this recommendation implies that programs launch their students on a trajectory of continued professional growth, building "their emerging leadership by being capable and effective advocates for excellence in mathematics teaching and learning and by holding themselves and their colleagues responsible for the mathematical success of all students" (MTE-Partnership, 2014, p. 5).

Methods Courses

Programs preparing high school mathematics teachers should provide multiple opportunities to learn how to teach mathematics effectively through the equivalent of two mathematics-specific secondary-level methods courses, with at least half of the secondary level experiences targeted at the high school level. Methods courses should be offered as part of a broader program that includes other secondary-focused courses and high-school field experiences. [Elaboration of P.3.4]

Mathematics methods courses must address deep and meaningful mathematics comprised of the interrelationship among concepts, procedures, reasoning and justification, problem solving, and developing productive dispositions (Kilpatrick, Swafford, & Findell, 2001). Teacher preparation programs that have adopted the recommendations in this document will seek to prepare candidates by revisiting high school mathematical content from a rich and meaning-making way, but even the three content courses particularly addressing the needs of prospective high school mathematics teachers recommended in the elaboration of Standard C.1 will not overcome the influence of a preservice secondary mathematics teacher's own K-12 learning experience if it was focused primarily on procedures. Furthermore, given the variety of paths taken by students prior to credential programs, some students in mathematics methods courses may have completed fewer than three specially-designed mathematics courses for prospective high school teachers. As such, mathematics methods courses must focus not only on the methods of teaching mathematics, but also on the *high school mathematics* for which those methods are intended to teach.

Teachers must also be prepared to hold strong foundations of knowledge about students as mathematics learners, including knowing about research-based progressions within well-defined content domains, understanding students' ways of engaging in mathematical practices, helping students share their mathematical thinking and leveraging the diversity of students' thinking to advance instruction. Consequently, experiences for prospective secondary school teachers to investigate high school students' mathematical thinking should be incorporated into mathematics methods courses and early field experiences.

Mathematics methods courses must also address the social, historical, political and institutional contexts that affect mathematics teaching and learning and provide practice-based experiences to develop core practices and pedagogical content knowledge that honors mathematics, students' mathematical thinking, and cultural/community-based funds of knowledge and experience.

As important as it is for prospective teachers to grapple with issues of mathematics, students' thinking, and equity, the experience is more valuable for prospective teachers when these issues arise simultaneously. How might mathematics methods instructors address issues of equity alongside issues of mathematical content and students' thinking? Consider an example by Civil (2016) of a prospective teacher who, after reading an article about algorithms taught in Latin American countries that differed from those taught in U.S. schools, said, "This is nice, but they need to learn to do things the U.S. way" (Civil, 2016, p. 220.) This case raises a myriad of issues, including valorization of knowledge, whereby the knowledge held by immigrant children is not valued as much as the knowledge that is taught in U.S. schools, the role of convention and principle in algorithms, and the great pedagogical value in understanding and building upon students' mathematical thinking. All these issues must be given space to arise and be discussed in mathematics methods classes.

Standard P.4. Opportunities to Learn in Clinical Settings

Perhaps central to successful clinical experiences at the high school level are mentor teachers who themselves reflect the values discussed in Chapter 2. These mentor teachers need to support candidates in overcoming their possible inherent belief that “teaching as they were taught” will work for their students since it worked for them. Mentor teachers need to help candidates realize the limitations of typical instructional approaches in supporting the learning of all students. They need to support the goals of the program so that there is a continuity between experiences provided in university classrooms and what they experience in their clinical settings, rather than the dismissive mantra, “Don’t listen to those people in the ivory tower.”

While many high school mathematics teacher candidates may have chosen their career based in large measure on their enjoyment of mathematics, they need to build a parallel enjoyment in helping all students to come to understand and appreciate mathematics, not just the students who have already built an enjoyment for doing mathematics. Clinical experiences should be carefully constructed to help candidates move beyond a focus on managing a classroom to truly engaging all students in doing mathematics.

Candidates need to be supported in navigating the diverse culture of high school mathematics teachers. Some who are committed more to teaching mathematics than teaching children, some who are only a few years older than the student they are teaching, all who are encountering the challenges of changing the very entrenched vision of “high school” into a more engaging, collaborative, interdisciplinary educational setting, while wrestling with traditional expectations on the part of parents, policymakers and other about what high school should be. The inherent changes and diverse cultures require communities in which high school mathematics teachers can learn from each other and improve through collaboration.

The clinical experiences of effective teacher preparation programs are guided by a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for teacher candidates to teach and to learn from their own teaching and the teaching of others.

Indicators include:

- P.4.1. Collaboratively develop and enact clinical experiences
- P.4.2. Sequence school-based experiences
- P.4.3. Experience teaching with diverse learners
- P.4.4. Recruit and support qualified mentor teachers and teacher preparation supervisors

Standard P.5. Recruit and Retain Teacher Candidates

Given the critical shortage of well-prepared teachers of high school mathematics, it is imperative that effective high school mathematics teacher preparation programs devote significant attention to attracting talented mathematicians into teaching, particularly given the perception that teachers receive minimal pay relative to other professions that individuals with a baccalaureate in mathematics can pursue. It is also particularly hard to recruit individuals of color into teaching, again given the allure of more high paying and prestigious jobs that are also seeking to diversify their workplaces.

An effective mathematics teacher preparation program attracts, nurtures, and graduates high quality teachers of mathematics who are representative of diverse communities.

Indicators include:

P.5.1. Recruit strong candidates

P.5.2. Address diverse community needs

P.5.3. Provide experiences and support structures

P.5.4. Assess recruitment and retention data

The traditional pathway into mathematics teaching by those who love doing mathematics is still present, and effective programs should capitalize on that. But there may be other pathways that can be explored; for example, programs might appeal to those who find enjoyment in helping others. Efforts must also be made to counter the narrative that high school teaching jobs are particularly low paying; some research is emerging that shows a significant difference between what STEM majors perceive to be teacher salaries and actual salaries. While the pay may be low, it is not as low as perceived by those who might consider a career as a mathematics teacher (Dickey, 2016).

Potential Pathways for Preparing Secondary Mathematics Teachers

Programs to prepare secondary mathematics teachers can be organized in many different ways, depending on state policies, college or university guidelines, the intended audience (e.g., career-changers), and so forth. However they are organized, effective programs must meet the requirements of these standards in this document. Following are several examples of how differently-organized programs can meet the standards.

Program 1. Four-year Bachelor of Science Degree

One approach to establishing a program that leads to the well-prepared beginning teacher of mathematics could be in the form of a four-year Bachelor of Science degree. This program has a major in mathematics with a teaching option. Consistent with MET2 (CBMS, 2012), mathematics and statistics coursework consists of single and multivariable calculus, differential equations, two courses in data-based statistics and statistical inference, transition to proof, and linear algebra, each at the lower division. Upper division coursework includes three courses designed specifically for teachers: one course focused on number theory, algebra, and the real number system and functions; the second course focused on Euclidean and non-Euclidean geometry; and a third course focused on modeling using the tools of mathematics without calculus and using simulation-based statistics. Each of these courses embeds the use of technology as a tool for learning mathematics. Mathematics coursework also includes three upper division elective courses in mathematics or statistics, and students choose from among courses including algebraic or geometric reasoning in the middle grades; history of mathematics; and others.

The program includes designated mathematics methods courses with field experiences, and embeds selected opportunities for working with students within mathematics content courses. While the ideal would provide three methods courses, this is not currently possible given the constraints of the program

requirements as a whole. The program designers recognized this need for attention to teaching methods, and have addressed it by integrating pedagogy assignments and field experiences within mathematics content courses. The program includes education coursework taken by all secondary teaching majors in the state, and includes student teaching.

Program 2. Fifth-year Program

A second program achieves the recommendations of this chapter via a fifth year program that follows a strong undergraduate major in mathematics teaching. The undergraduate coursework includes three content courses addressing mathematical content for teaching. In addition to generic education coursework taken by all secondary teaching majors, two content-specific secondary mathematics methods courses, both of which include explicit attention to the teaching and learning of high school mathematics topics, are required. Furthermore, although equity issues are addressed in generic education courses, equity issues must also be explicitly addressed within mathematics methods courses, because only then will candidates understand that equity is not separate from, but fundamentally a part of, what it means to effectively teach mathematics to high school students. Student teaching takes place concurrently during the courses, with the student assuming increased classroom teaching responsibility over the course of each semester and from the first to the second semester. Because student teaching occurs concurrently with taking courses, issues that arise in student teachers' practice are incorporated into the mathematics methods course discussions. Note that students coming from a general mathematics major that does not focus on mathematics teaching may have to complete pre or co-requisite coursework.

Program 3. Liberal Arts Program

In a third program, the recommended standards are achieved by a liberal arts mathematics program that is a part of a coalition of universities and colleges offering secondary mathematics teacher preparation. Specialized mathematics content courses for teachers, as well as mathematics-specific methods courses, are offered collaboratively with the nearby state university that leads a local partnership team. Some of these jointly-offered courses are delivered online. In other cases, students meet on one of the campuses or at a central location. While this allows students in the smaller program to complete the requirements for becoming a secondary mathematics teacher, it also provides an opportunity for students at the state university to interact with colleagues from another context, thus broadening their awareness of issues related to mathematics education, particularly if the liberal arts college reflects different demographics from the state university.

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CHAPTER 8. ASSESSING MATHEMATICS TEACHER PREPARATION

Assessment of the preparation of mathematics teachers is purpose-driven. Multiple purposes must be served, including but not limited to:

Inform progress of candidates towards becoming well-prepared beginning teachers of mathematics	Determine candidates' readiness to begin practice as beginning mathematics teachers (credentialing or licensing)
Understand a program's strengths and weaknesses to improve the program	Meet standards for program accreditation

These purposes are served through sound and efficient assessments capable of securing information, supporting interpretation, and taking action. Assessments necessarily appraise mathematics teacher candidates and elements of the mathematics teacher preparation programs that support their professional growth, providing timely formative information needed to provide ongoing feedback to candidates and to tailor learning experiences that support their growth. Assessments also provide summative information on the outcomes of teacher preparation, most crucially providing information on the quality of the knowledge, skills, and dispositions of mathematics teaching candidates and the candidates' proficiency in supporting mathematical learning, as well as overall program quality.

As an organization, AMTE is committed to promoting the improvement of mathematics teacher preparation, which is also the goal of this document. **Therefore, this chapter focuses on formative, improvement-oriented uses of assessment that are aligned with the teacher preparation standards described in earlier chapters.** This document sets forth standards for what candidates should know and be able to do, as well as what programs need to do in order to support their candidates in achieving those standards. To be effective, programs need assessments that provide useful information about their candidates, both individually and collectively, in attaining those standards, to identify areas where programs need to improve, and to better support their candidates' growth towards becoming well-prepared beginning mathematics teachers.

While we choose to focus on the use of these standards for purposes of ongoing improvement, we acknowledge that the standards could at some point and in some ways inform summative purposes. For example, these standards are not designed to establish minimum benchmarks that candidates need to meet before beginning their careers as mathematics teachers. However, these standards provide useful insights about important areas that should be considered in devising such benchmarks. Likewise, the emphasis on program improvement is consistent with Standard 5 of the accreditation standards put forward by the Council for the Accreditation of Educator Preparation (CAEP).

This chapter begins with recommendations about general qualities of assessment used in mathematics teacher preparation; these recommendations are abbreviated AQ. It then discusses the qualities of assessments that align with the vision of candidate quality (abbreviated AC) and with overall program quality (abbreviated AP) embodied in these standards in order to effectively guide efforts to improve mathematics teacher preparation. A summary of the recommendations follows in Table 8.1.

Table 8.1. Recommendations about Assessing Mathematics Teacher Preparation

Qualities of Assessments

Recommendation AQ.1: Focus on mathematics teaching

Assessments are tailored to provide information relevant to developing effective teachers of mathematics.

Recommendation AQ.2: Promote equity and access

Assessments are designed, implemented, interpreted and used to advance equity and access.

Recommendation AQ.3: Embody openness

Assessment is a transparent process in which those assessing, those being assessed, and other stakeholders know the assessment focus, the nature of assessment methods and criteria, and the uses of the information gathered.

Recommendation AQ.4: Support valid inferences and action

Assessments appropriately include multiple measures that are well-suited to gather the type and amount of information needed to make valid inferences and take action.

Recommendation AQ.5: Embody coherence and sustainability

Assessments are aligned with teacher preparation goals, opportunities to learn, and intended uses of information. They are designed and enacted in ways that can be sustained over time.

Assessing Mathematics Teacher Candidate Quality

Recommendation AC.1. Mathematical knowledge for teaching

Assessments of mathematics knowledge need to incorporate attention to candidates' development of mathematical knowledge for teaching, including processes and practices.

Recommendation AC.2. Assessment of mathematics instructional practice

Assessments of mathematics teaching practice should include observations of teaching that focus on how well it supports the learning of important mathematical content and processes and practices by all students.

Recommendation AC.3. Assessment of dispositions

Assessments should include emphasis on a range of dispositions related to mathematics teaching, including dispositions towards doing mathematics, identity as a mathematics teacher and learner, and commitment to support the mathematics learning of all students.

Assessing Mathematics Teacher Preparation Program Quality

Recommendation AP.1. Stakeholder engagement

Effective mathematics teacher preparation programs should assess the degree to which a broad range of stakeholders are meaningfully engaged in the program and involved in its ongoing improvement.

Recommendation AP.2. Program curriculum and instruction

Effective mathematics teacher preparation programs should assess the effectiveness of the courses they offer in promoting their candidates' progress and work to improve them.

Recommendation AP.3. Effective clinical experiences

Effective mathematics teacher preparation programs should actively assess and work to improve the quality of the clinical experiences they provide mathematics teacher candidates.

Recommendation AP.4. Recruitment and retention

Programs must gather data to assess both recruitment efforts and successful retention by tracking the intake of candidates, monitoring candidates' progress through the program and to program completion, and tracking early career teachers who continue to work in the education profession, including considerations of demographics reflective of the student population of the region.

Qualities of Assessments Used in Mathematics Teacher Preparation

There is no shortage of guidance in research literature and professional standards on the traits of sound assessments. In the case of assessments that will be generative for mathematics teacher preparation, literature and standards focused on teacher education and mathematics education are certainly useful. One source that was particularly pivotal in our consideration of the recommendations below was the National Council of Teachers of Mathematics *Assessment Standards for School Mathematics* (NCTM, 1995). Twenty years have passed since the release of those standards. Further, the standards presented in that document were focused on assessment in K-12 mathematics. However, those standards continue to embody traits needed for high quality assessment in mathematics teaching and learning.

The recommendations on the qualities of assessments provide a set of criteria by which individual and systems of assessments can be judged. More generally, they relate to the content and forms of assessment, as well as routines that support the implementation of assessments and uses of assessment information. This means that the recommendations can be used proactively as goals for assessment selection and development. They can also shape dispositions guiding implementation of assessments.

Recommendation AQ.1: Focus on mathematics teaching

Assessments are tailored to provide information relevant to developing effective teachers of mathematics.

Assessments in mathematics teacher preparation capture key knowledge, skills, and dispositions well prepared beginning teachers need to support mathematics learning. Assessments at the program level must facilitate the capture and aggregation of information about mathematics teaching. Two components of assessment practice are entailed in the focus on mathematics teaching. First, it is important that assessments are sensitive to the *mathematical* aspects of teaching, as opposed to reliance on generic or subject matter neutral approaches. Second, while it is possible for programs to assess aspects relevant to mathematics teaching outside of practice, it is crucial that they use assessments capable of appraising candidates' *engagement in mathematics teaching*, including supporting candidates' self-assessment of their own mathematics teaching. Given the complexity of mathematics teaching, those preparing future mathematics teachers often engage candidates in approximations of teaching that allow for some aspects of teaching to be more authentic while

temporarily suspending other aspects. Assessments can and should happen in these contexts to provide information on growth that can be used to shape candidates' subsequent opportunities to learn about mathematics teaching.

Recommendation AQ.2: Promote equity and access

Assessments are designed, implemented, interpreted and used to advance equity and access.

Assessments in mathematics teacher preparation must allow all candidates equitable opportunities to demonstrate their knowledge, skills, and dispositions. This includes access for all candidates to assessments that allow them to show their ability to teach mathematics and to receive generative, mathematics teaching-focused feedback on their teaching. Criteria defining mathematics teaching proficiency honor diverse approaches in the service of students' meaningful and productive engagement with mathematics. All assessments are routinely analyzed for signs of bias or patterns of outcomes that indicate that the assessment is functioning inequitably. Results of assessments should lead to the improvement of opportunities to learn about mathematics teaching, including differentiation of supports and resources to enhance the performance of each and every candidate. The tools used should produce reliable results and be valid in what they measure.

Recommendation AQ.3: Embody openness

Assessment is a transparent process in which those assessing, those being assessed, and other stakeholders know the assessment focus, the nature of assessment methods and criteria, and the uses of the information gathered.

Transparency in assessment means that all involved in assessments and those stakeholders who use information from assessments are fully informed and also have suitable roles in assessment selection, implementation, and the use of assessment information. The goals for assessments should be clear to all involved, and the assessments should be validated by those involved as providing useful and actionable information. It is particularly important that the assessment process be open to mathematics teacher candidates. This can be accomplished in many ways, including through an orientation to build self-assessment components into most assessments and involving candidates in the development of assessment criteria. Candidates and other users of assessment information should receive performance information in timely ways and in forms that are easily interpreted and meaningful. While there are times when some aspects of assessments need to be secure, mathematics teacher preparation programs are vigilant to make sure that this practice is limited and routinely reconsidered for opportunities to enhance the openness of the process and content of assessments.

Recommendation AQ.4: Support valid inferences and action

Assessments appropriately include multiple measures that are well-suited to gather the type and amount of information needed to make valid inferences and take action.

Assessments of mathematics candidate and program quality should be comprehensive, utilizing multiple measures that address the range of relevant standards. Relying on single data point in required assessments (such as a PRAXIS score) will not provide the breadth of information that is needed. Assessments must specifically address mathematics teaching; general measures will not provide adequate information about their readiness to teach mathematics. The assessments cannot focus solely on candidates' mathematical knowledge, but must include attention to their instructional practices and dispositions related to mathematics, as outlined in Chapter 2.

Assessment approaches should be chosen for their capacity to capture information most relevant to the knowledge, skills, and/or dispositions targeted in the assessment. Ease of administration must not be the leading consideration when assessment methods are chosen, just as assessment targets should be chosen by their importance not in light of the ease of assessing them. Further, many valued outcomes are not easily or well conveyed quantitatively. Questionnaires, interviews, focus groups, observational notes, video recordings of teaching, and other records of practice provide critical complementary data. Although the learning of mathematics achieved by their students may be the ultimate assessment of a candidate's quality, measures of student success that focus on a single high-stakes assessment are likely not valid and are unlikely to provide information that can fuel improvement.

Recommendation AQ.5: Embody coherence and sustainability

Assessments are aligned with teacher preparation goals, opportunities to learn, and intended uses of information. They are designed and enacted in ways that can be sustained over time.

Mathematics teacher preparation assessments are logical outgrowths of program goals and learning experiences. This coherence makes the content within assessments and approaches to assessment seem natural and expected - instead of surprising or worse yet unfair or alarming. Further, the strong connection also positions users of assessment data to see the ways in which insights relate back to learning opportunities and project forward into action.

Assessments in mathematics teacher preparation need to be chosen, enacted, and used in ways that can be replicated over time, not simply at a particular point in time. Routinely, efforts are made to enhance sustainability of the assessment system. However this does not mean that assessments within the system are lightly or frequently changed. Attending to sustainability in these ways makes it possible to collect the data necessary for mathematics teacher preparation programs to answer important longitudinal questions about course and program effectiveness, the nature and degree of consensus among and between candidates, mentor teachers, and program instructors with respect to mathematics teaching quality, and the correlation or at least correspondence between particular assessments used by programs.

Assessing Mathematics Teacher Candidate Quality

Assessing candidate quality is foundational to preparing mathematics teachers. The assessment qualities defined above must be taken into consideration when assessing candidate quality. Assessments must be selected or developed that provide insight into the knowledge skills and dispositions of the mathematics teacher. This requires that assessments are sensitive to the mathematical and pedagogical dimensions of quality mathematics teaching. Assessments must be accessible and promote equity, both in the sense of equitably appraising the mathematics teacher candidates themselves and assessing the ability of candidates to engage in teaching practices that promote the mathematical achievement of all their students. Every mathematics teacher candidate should be fully aware of the assessments, products, criteria, and potential consequences of the assessments in which they engage. Those assessing the quality of mathematics teacher candidates must draw on assessments and develop systems that are valid, coherent, and sustainable.

There are many ways in which assessments of candidate teacher quality can be used to improve mathematics teacher preparation. Candidates can be positioned to learn from video and other records of their own teaching experiences and to apply criteria reflectively to their performances. Mentor teachers and field supervisors can analyze appraisals of teacher candidates' teaching with an eye toward

providing opportunities to further hone particular teaching practices or to expand knowledge of children. Course instructors can look across reflections of teacher candidates to notice areas where they could raise awareness of bias that might be interfering with their ability to formulate productive next steps in teaching. Content course instructors could look at student work that is collected by teacher candidates to prioritize which mathematical understandings, principles, or practices to bring to the fore. Program administrators could look at the performance of teacher candidates at a particular point in the program and marshal resources that could expand or deepen work on a particular teaching practice.

Table 8.2 provides an overview of potential attributes and measures of interest to mathematics teacher educators. Programs should consider what collection of measures provide the most meaningful data regarding candidate's mathematical knowledge, instructional practice, and dispositions based on the values and goals of the program.

Table 8.2. Attributes Related to Mathematics Teacher Preparation Quality and Evidence Used to Measure Them

Attribute	Measures
Admissions and recruitment criteria	<ul style="list-style-type: none"> • GPA • Average entrance exam scores (e.g., basic skills exam, PRAXIS multiple subject or mathematics exams) • Number and diversity of candidates • Dispositions self-assessment and feedback from recommenders
Quality and substance of instruction	<ul style="list-style-type: none"> • Course syllabi and textbooks • Course offerings including mathematics courses • Course common assessments with common rubrics measuring both mathematical content knowledge and ability to engage in mathematical practices measuring candidate progress and growth over time • Observations/video recordings to support instructor reflection
Quality of clinical experiences	<ul style="list-style-type: none"> • Fieldwork policies, including required hours • Qualifications of fieldwork mentors • Surveys of candidates • Field evaluations that are aligned with effective mathematics teaching practices • Data regarding candidate impact on student understanding and achievement (e.g., equitable student learning across various student groups) • Dispositions self-assessment and feedback from field mentors
Faculty Qualifications	<ul style="list-style-type: none"> • Percentage of faculty with advanced degrees • Percentage of faculty that are full-time, part-time, adjunct
Effectiveness in preparing new teachers who are employable and stay in the field	<ul style="list-style-type: none"> • Exit and alumni survey on preparedness • Scores from field evaluation data during program and teacher evaluation once in the field • Hiring and retention rates

Success in preparing high-quality teachers	<ul style="list-style-type: none"> • Teacher performance assessments • Ratings of graduates by principals/employers • Value-added estimates
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Adapted from Feuer, M. J., Floden, R. E., Chudowsky, N., and Ahn, J. (2013). *Evaluation of teacher preparation programs: Purposes, methods, and policy options*. Washington, DC: National Academy of Education.

These examples only scratch the surface of what is possible. This is not meant to imply that the focus should be on the number of assessments used. Rather, the examples convey that there is an array of assessment data that could be relevant, contexts in which assessments may occur, and people who could beneficially be involved in such assessments. The recommendations below surface implications of the standards for well-prepared beginners articulated in chapter 2 on assessment of mathematics teacher candidates aimed at improvement. The text for each recommendation elaborates key considerations of assessment and also illustrate modes and contexts of assessments, facilitator of assessments, and crucially point toward uses of assessment information.

Recommendation AC.1. Mathematical knowledge for teaching

Assessments of mathematics knowledge need to incorporate attention to candidates' development of mathematical knowledge for teaching, including processes and practices.

Programs include a variety of assessment methods to assess the depth of content knowledge and knowledge and use of mathematical practices and processes of prospective teachers. Effective programs use more than tests embedded in coursework or an external test required to meet accreditation to assess prospective teachers' knowledge of mathematics. Courses include assessment strategies in alignment with approaches that prospective teachers will be expected to enact as future teachers (cf. NCTM, 2014). Assessments should address not only their ability to produce correct answers, but also their ability to reason, explain, and engage in other mathematical practices. Mathematicians are increasingly attending to the disconnect between what is valued and what is assessed (Tallman, Carlson, Bressoud, & Pearson, 2016). Measures other than tests should be used, such as projects and performance-based assignments. Self-assessment should also be a regular part of coursework, and formative assessment should be an integral part of instruction and be used to make instructional decisions (AMTE/NCSM, 2014).

Beyond assessment within courses, it is important for programs to assess candidates' progress across courses. Quality assessment is more than an accumulation of individual course grades, and includes an assessment of dispositions, agency, and understanding of the nature of mathematics, consistent with standards C.1.3, C.2.1, and C.1.5. Although external tests may be a required part of certification, programs must include other measures in assessing the progress of prospective teachers in learning mathematics. For example, tests of mathematical knowledge for teaching--such as the Learning for Mathematics Teaching (2011) assessment or the Diagnostic Mathematics Assessments for Middle School Teachers (Bush et al., 2005)--or assessments of dispositions towards mathematics might be incorporated at critical points in the program or as an exit requirement prior to student teaching. A high quality program might also use a candidate-created portfolio as an overall assessment of the depth and soundness of their knowledge of mathematics content and use of mathematical practices and processes.

Recommendation AC.2. Assessment of mathematics instructional practice.

Assessments of mathematics teaching practice should include observations of teaching that focus on how well it supports the learning of important mathematical content and processes and practices by all students.

Programs best serve candidates when candidates receive regular ongoing feedback from mentors and supervisors who know and have taught mathematics. These observers benefit from training and calibration using tools that are specific to the effective planning, teaching, and assessment practices highlighted in standards C.2, such as the Mathematics Classroom Observation Protocol for Practices (Gleason, Livers, & Zelkowski, 2015), Reformed Teaching Observation Protocol (Sawada & Piburn, 2000), and Student Discourse Observation Protocol (Weaver, Dick, & Rigelman, 2005). The observation cycle is most powerful when it includes pre-observation discussion, focused observation a lesson, data-based discussion following the lesson, and follow-up that holds candidates responsible for and supports their continued growth in meeting the needs of each student. The pre-observation can engage the candidate with articulating their learning goals for students as well as potential foci for the observation (e.g., student engagement at various points in the lesson, responsiveness to student thinking/ideas, teacher questioning and wait time, student mathematical discourse, student use of mathematical tools). The debrief of the lesson uses the data gathered by the observer to inform progress toward the learning goals and well as growth in instructional practice. Finally, following the observation cycle, the observer follows up with the candidate by checking on progress and/or providing resources to support continued development of instructional practice.

Recommendation AC.3. Assessment of dispositions.

Assessments should include emphasis on a range of dispositions related to mathematics teaching, including dispositions towards doing mathematics, identity as a mathematics teacher and learner, and commitment to support the mathematics learning of all students.

Although candidate reflection on their dispositions can provide useful information, and can provide a starting point for their continued growth, assessments of dispositions need to rely on more than self-reports. The self-report data can be powerfully combined with reports from those working most closely with the candidate (i.e., advisers, faculty, mentor teachers, supervisors) and can serve as a point of reflection and growth for the candidate. Dispositions can be further examined through candidates reflecting upon ways that they are supporting their students with developing productive identities as mathematicians.

Assessing Mathematics Teacher Preparation Program Quality

Although the success of a mathematics teacher preparation program ultimately rests on the progress of its candidates, assessment of particular program features can provide important insights into how the program can improve. This section describes how the standards from Chapter 3 might be assessed in order to guide those preparing mathematics teachers as they seek to improve their programs. Mathematics teacher preparation program evaluation should be dominated by high-quality, valid assessments where results are used to spur and inform continuous program improvement. Programs can achieve such ongoing review by regularly collecting, disseminating, and using data in a timely way to prompt conversations among stakeholders in order to consider strengths and potential areas for growth.

Recommendation AP.1. Stakeholder engagement

Effective mathematics teacher preparation programs should assess the degree to which a broad range of stakeholders are meaningfully engaged in the program and involved in its ongoing improvement.

Effective mathematics teacher preparation programs require the dedication and commitment of all those involved in preparing candidates. As described in Standard 3.1, while leadership for creating and sustaining the program will typically fall to mathematics teacher educators, a range of other stakeholders must be engaged, including colleagues in teacher preparation, P-12 educators and administrators, mathematicians, and community-based organizations and community members.

Programs might use surveys, individual interviews, or focus groups to gather information from a range of stakeholders to answer questions such as the following:

- To what degree do the stakeholders believe that they are an important part of the mathematics teacher preparation program, that their perspectives are heard and acknowledged?
- To what degree do they perceive that mathematics teacher preparation program is an important and meaningful part of their job, that their efforts contribute to the development of quality teacher candidates?
- To what degree do they feel an authentic partnership exists among the stakeholders, that they are not merely service providers to the program but an integral part of the system?
- To what degree are the goals and values promulgated by the program shared across all those involved in the program?
- What areas of the program do they feel are functioning well? What areas need additional attention?

It is also crucial that mathematics teacher preparations gather and use data that is valued by all stakeholders to guide efforts to improve the program. This data should be collaboratively considered to develop priorities where improvement is needed, as well as to guide improvements in particular areas of the program.

Recommendation AP.2. Program curriculum and instruction

Effective mathematics teacher preparation programs should assess the effectiveness of the courses they offer in promoting their candidates' progress and work to improve them.

While the typical course evaluation procedures used by colleges and universities can provide limited information about the effectiveness of courses included in the mathematics teacher preparation program, effective mathematics teacher preparation programs need to undertake a deeper assessment to ensure that all such courses—including methods courses, mathematics content courses, and foundational courses—positively contribute to the growth of their candidates.

First, all courses should model effective instructional approaches aligned with those programs want candidates to develop. Instructors should be encouraged to individually self-assess their instruction and to collaborate with others about how instruction in key courses for mathematics teacher candidates can be improved. Peer observations using adaptations of observation protocols used to observe the teaching of candidates might provide a useful starting point in such deliberations. For example, the Mathematics Classroom Observation Protocol for Practices (Gleason, Livers, & Zelkowski, 2015) has been validated for mathematics classrooms K-16, and an adaptation for methods courses might be explored.

Second, the content of courses in the program should be analyzed to be sure they promote candidate growth. Course objectives and assignments should be examined to ensure coherence across the program. Mapping the experiences provided onto the standards in this document, particularly P.2 and P.3, may help to identify areas where more attention needs to be provided or areas of overlap.

Finally, as stated in Standard P.3.5, programs should assess whether instructors of mathematics methods courses have relevant grade-level experiences needed to support their candidates' growth. While instructors of mathematics content courses need not have this same level of experience, they should be responsive to the content needs of mathematics teacher candidates, as outlined in Standard P.2. Instructors of other foundational courses in education should be aware of the standards in chapters 2 and 3 relevant to their course content.

Recommendation AP.3. Effective clinical experiences

Effective mathematics teacher preparation programs should actively assess and work to improve the quality of the clinical experiences they provide mathematics teacher candidates.

In assessing the quality of the clinical experiences they provide their teacher candidates, effective mathematics teacher preparation programs should consider issues such as the following which are identified in Standard P.4:

- Engagement of mentor teachers in enacting a shared vision of quality mathematics instruction. Effective programs need to assess what mechanisms are in place for ensuring bi-directional discussions with mentor teachers about mathematics teaching and learning. They need to engage mentor teachers in discussions about how clinical experiences can be better organized in order to drive continued improvement of the experiences offered.
- Effective sequencing of experiences. This might include examining requirements for experiences to ensure that they are coherently organized to respect the candidates' development and to become increasingly comprehensive in scope.
- Range of experiences. Records of the experiences candidates have should be analyzed to ensure a range of grade levels that reflect their certification level, as well as with students of different backgrounds.
- Qualifications of mentors. Programs need to assess their selection and use of mentor teachers and university supervisors to be sure they reflect program values, demonstrate effective teaching, and have effective mentoring skills.

Evidence should also be gathered about the overall effectiveness of clinical experiences. This might include looking at candidates' success in the program, surveys of both students and mentor teachers, and ratings of candidates on observation protocols. These analyses lead to discussions among university and school personnel about areas where clinical experiences support candidate growth and areas where adjustments may be needed.

Recommendation AP.4. Recruitment and retention

Programs must gather data to assess both recruitment efforts and successful retention by tracking the intake of candidates, monitoring candidates' progress through the program and to program completion, and tracking early career teachers who continue to work in the education profession, including considerations of demographics reflective of the student population of the region.

Effective mathematics teacher preparation programs must proactively track their ability to attract and produce well-qualified candidates so as to ensure that programs are meeting the needs of schools in their region, including reflecting the demographics of those schools. This begins with a consideration of the quality of candidates admitted to the program, along with their diversity, and requires direct strategies tied to recruitment. Over-reliance on measures such as GPA or entrance tests may unjustly eliminate qualified candidates, given that there is little evidence to suggest that such measures are connected to later success as a mathematics teacher. Consideration of other factors, such as flexibility in thinking and being able to relate to others unlike oneself, may in fact better predict a candidate's potential to learn to be a teacher and should be explored as possible measures. Gathering data about candidates and tracking their success in the program and into their early careers provide valuable information about program quality and justification of admission and program completion decisions.

Effective programs must also proactively track the growth of their candidates across the program, particularly working to identify potential hurdles or barriers and considering how they can be addressed. Candidate input should be sought on how the program can better support their progress, and all those involved in preparing candidates should work collaboratively to ensure that focus is maintained on supporting the development of well-prepared beginning mathematics teachers.

Further recommendations about creating systems to track recruitment and retention data are given in Indicator P.5.4 in Chapter 3. We note in particular, "Educator preparation programs must also monitor the early career progress of their graduates to inform program improvement as well as to contribute the retention of teachers."

Conclusion

The standards in this document set an aggressive target for the knowledge, skills, and dispositions of well-prepared beginning mathematics teachers, as well as the qualities of effective mathematics teacher preparation programs. They are intended to guide efforts to improve mathematics teacher preparation programs. However, improvement is not possible without assessment. Evidence is necessary in identifying areas where improvement is needed. And it is needed to guide the the process of improvement; as the old saying goes, not every change is an improvement. Thus, seeking effective ways to assess the quality of candidates and of the program itself are central to attaining the goals of this document.

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