**Developing the Ability to Respond to Student Thinking**

*Goals for this assignment:*

1. Develop a variety of strategies (e.g., alternate tasks, review, questioning) for immediately responding to students’ thinking in order to further their understanding of a mathematical concept.
2. Become aware of the power of carefully chosen questions in eliciting and developing students’ mathematical thinking and reasoning as well as moving students to a higher level of understanding.
3. Practice developing questions that will help elicit and develop students’ mathematical thinking in response to student work.
4. Consider the importance of holding back from telling students something they could work out for themselves.
5. Reflect on ways of responding to students’ thinking.

*Student learning objectives for the task:*

1. Students will be able to solve systems of equations through multiple representations.
2. Students will be able to explain in multiple ways (verbally, symbolically, and graphically) the significance of having two expressions set equal to one another and the meaning of the solutions. (Students may also use a table to explain their thinking)

*The assignment:*

1. In small groups, discuss the student responses to the tasks from your interviews (referencing the transcripts and student work from the paper you wrote) then answer the following questions.
* Were there instances where you wanted to ask the student follow-up questions to help you understand what the student was thinking? Describe those instances.
* While the original interview was observational only (no teaching was to be done), were there instances where you wanted to respond to the student from a teaching perspective, advancing the student towards understanding how to solve these equations in multiple ways? Describe those instances.
1. Given the following work from Taio (a pseudonym, as are all student names) and statements from when Taio solved problem B, write your responses to the following prompts:



**“Since there's parenthesis I multiply. Then move the x's so they're on the same side. They cancel out so 8=8, which is true.”**

* + What possible interpretations are there of Taio’s statement “They cancel out so 8=8, which is true”?
	+ Do you think Taio has solved this problem correctly? Explain.
	+ What questions would you ask Taio to help you understand what this student is thinking?

**Clarifying questions** can be used to learn more about how a student was thinking and is an appropriate response in situations where a teacher needs more information in order to respond. Here are some clarifying question prompts that can work well for this:

* How did you get your answer?
* What do you mean by [restate the portion of the response you need clarification about]?
* Can you explain why you are doing that?
* What were you thinking about when you solved this problem?
* I noticed you did [state student action]. Why did you do that?

**Responding**

Once you have the information you need to interpret a student’s work, you should respond to the student. The literature (Jacobs, V. R., Lamb, L. L., Philipp, R. A., & Schappelle, B. P. (2011). Deciding how to respond on the basis of children’s understandings. *Mathematics teacher noticing: Seeing through teachers’ eyes*, 97-116) suggests four characteristics of a good response to students:

**Characteristics of a good response**:

1) Works towards student learning objective.

2) Draws on and is consistent with the student thinking presented.

3) Draws on and is consistent with research on students' mathematical development.

4) Proposed interaction with student leaves space for student's future thinking (not just teacher's thinking)

Things to think about as you develop your response:

* + Are there *specific questions* that would move this student along?
	+ Are there *tasks* that could encourage new understanding or deeper thinking?
	+ Are there *review questions or tasks* that may be essential to this student understanding the concepts?
	+ What else would you do to help this student meet the given objectives?
1. Read the document “Developing Mathematical Thinking with Effective Questions” on page 8. This document provides questions that help develop students’ mathematical thinking and go beyond clarifying questions.
2. In the actual interview of Taio, the interviewer followed up this student’s statement with the clarifying question “True how?” which led to the following interchange:

**Interviewer: *True how?***

**Student: 8 does equal 8, so it's true.**

**Interviewer: *What does it mean to be true?***

**Student: I dunno [don’t know].**

Imagine you are in the role of this student’s teacher. You’ve had this interchange with the student and now are ready to respond at this moment to the student’s statements.

Considering the additional information from the transcript of Taio’s thinking that was provided, the questioning prompts, and the criteria for a good response, how would you now respond to this student?

*Your response:*

1. Looking back at your response, consider the following questions and reflect on how you chose to respond in number 4.
2. Does your response help to elicit and develop children’s thinking with the learning objectives in mind? In what ways?
3. Does your response build on the student thinking that is evident in the written or transcribed work? How?
4. Does your response build on what you know about how we want students to think about solving linear equations (e.g., being versatile and adaptable among representations)? How?
5. Are you telling students something they could work out for themselves?
6. Revise your response from number 3, if warranted.
7. Below are 3 additional excerpts from previously conducted interviews with students that followed the Solving Equations protocol. For each, document your interpretation (or possible interpretations if the work is unclear) of the student work (written and/or transcribed statements) presented and develop a good response that you would provide to the student immediately following the student’s statement. Your response should be thorough enough to provide an opportunity bring to resolution the issue you note with the student’s understanding.

Recall that if you need more information in order to interpret the student’s work, a good response is a clarifying question to help you understand what the student is thinking. If you have enough information to interpret the response, a good response meets these 4 criteria:

1) Works towards student learning objective.

2) Draws on and is consistent with the student thinking presented.

3) Draws on and is consistent with research on students' mathematical development.

4) Proposed interaction with student leaves space for student's future thinking (not just teacher's thinking)

1. **Teresa [asked how to use the graph to solve problem B]:**

*Interviewer: Can you graph them?*

Teresa: Sure . . . this one doesn't give me another line, there's only one.

*Interviewer: What happened to the second line?*

Teresa: I dunno.

*Interpretation(s):*

*Your response:*

1. **Daniel [asked to solve problem B]**

Then, the next one, I’ll just use the distributive property first. So, 6x + 8 = 6x + 8. Already I can see that they’re the same, so x would just equal 1. But, um. So, just to solve it fully, minus 6x, subtract 8, subtract 8, 6x = 6x +0. And then if I were to divide by 6, divide by 6, x =1.



*Interpretation(s):*

*Your response:*

1. **Brendan [asked to solve problem B by graphing]**

For B, do I use this one or this one? [Pointing at the two equivalent sides]

*Interpretation(s):*

*Your response:*

1. Present your responses from #6 to the class. Compare and contrast your responses to those of your classmates. For each student, which response is most effective and why?



 