**Common Errors and Exemplars in PSTs’ Responses**

**on the Take Home Responding Assignment**

**Samples of Student Work Provided to PSTs:**

**SAMPLE A: MARK [solving problem B]**

Yeah, okay. So, for, so I want to get rid of this 2. That’s like on the side, so I’ll divide this whole other side by 2. I’m dividing the other side by 2 also to get rid of the 2. So then I get 3x plus 4 equals 3x plus 4. Because you divide 6 by 2 and 8 by 2. So then I can just subtract 4 from 4 which is 0 and subtract 3x from 3x which is zero. So… x = 0? Right? Cause you just divide 0 from 0, so x = 0?



**SAMPLE B: MARTHA [solving problem C]**

Um, so I wanted to get the x’s on one side and so I subtracted six x from both sides. Then I…then you had to get all of the constants on one side so I added five to both sides. Then I got eight plus five equals six x minus six x, and so I got thirteen equals x. 

**SAMPLE C: LAQUISHA [solving problem C]**

LaQuisha: So, I wouldn’t want to divide this by 2, cause 5 divided by 2 is kinda, kind of messy, so I think I’ll just do the distributive property again. 6x +8 = 6x-5, yup. And then, minus 8. Well, actually- if it’s a minus 8, I can just add 5, that’s easier. 6x +13 = 6x. Umm, minus 6x, minus 6x, so it’s just 13 = 0, so that’s…Either I did it wrong, or that’s just not equal to something. Or I could’ve…(concentrates)….Yeah.

Interviewer: So, is this your solution? (points at ‘13 =0’)

LaQuisha: Umm, kind of (LaQuisha then draws a line through the equals sign to indicate 13 does not equal 0).

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**SAMPLE D: TRISHA [solving problem B]**

Okay. Alright, so, I’m going to begin by… um… multiplying this parenthetical equation here, so it becomes…8…interesting. [laughs] Okay. Well, that’s obvious… [Looks at her result, which is 6x = 6x, for about 30 seconds.] Okay…Well, this one is confusing me, because they’re equal on either side now, so… um… x is… 1? Or…?... I don’t know. It’s fooled me.



**SAMPLE E: ZANDER [solving problem A]**

Interviewer: What are you thinking as you’re doing that part?

Zander: To get x by itself, but then there is another x over here.

Interviewer: So is x by itself?

Zander: This one is [points to left side of equation], but not this one

 [points to right side of equation].

Interviewer: Why not?

****Zander: Because it’s (5/2)x – 6 and I do

**4 Characteristics of a Good Response (used to assess PST responses)**

1) Works towards student learning objective.

2) Draws on and is consistent with the student thinking presented.

3) Draws on and is consistent with research on students' mathematical development.

4) Proposed interaction with student leaves space for student's future thinking

**Characteristics #1: Works towards student learning objective**

**Common Errors in Response to Mark’s Solution**

Many PSTs had difficulty providing responses that helped Mark work toward the learning objective. As illustrated in the following examples, PSTs often asked Mark questions but did not make sure he was able to achieve the learning objective through this interaction.

Student 1 Response

*-What does it mean for the two to be “on the side?” – I would like to hear a more clear explanation of what this means.*

*-Why did you subtract 4 from the left side and 3x from the right? – This may help him realize that he did not perform operations to both sides of the equation.*

*-You have written, “0x.” Explain how you got x=0 from this. –Trying to explain this step could lead the student to see that this is too big of a leap that does not make sense.*

*-Could you have solved this a different way?*

*-What does this tell you about the graph of this equation?*

Student 2 Response

*I would like to ask Mark what it means to subtract 3x from 3x, and how or why that is different from subtracting 4 from 4. These questions would be helpful because there seems to be a disconnect in how Mark understands numbers, and how he considers variables. This may stem partially from the fact that he felt like an x had to be in the final answer, whether that was a conscious decision or not. Perhaps these questions would begin to help him see that they are treated the same.*

**Exemplars in Response to Laquisha’s Solution**

These two responses show how PSTs are working toward the learning objective by getting the student to solve the problem using multiple representations.

Student 3 Response

*As her instructor I would ask “Can you explain what your solution means for the system of equations?” By asking her this I want her to understand that each solution or lack of a solution has a specific meaning for the system given. I would point to the line where she has 6x+8=6x-5 and ask her “What do you think about this? Can x be the same number?” When asking her about this line, I am highlighting the contradiction that is central to the system itself without telling her explicitly.*

*Because she doesn’t require any remediation of algebraic processes, I would move right to asking her about different representations of the problem. I would use the same line of questioning from the Huntley article and say “Can you solve the system in another way?” If she says no, I would ask her if she could graph both sides of the equation. Again, I am hoping to leave room for the student to make connections while guiding her towards the learning goals. Once she has graphed both sides of the equation, I would ask her “From the graph, can you explain the solution to the system of equations you solved for?”*

Student 4 Response

*--I would ask to explain what it means that 13* $\ne $*0. How would the graph of this look? What exactly is not equal?*

*--I had the learning objectives in mind when i asked her what the graph would look like. she is close to seeing the meaning of the solution but just required a little nudge in the right direction. by thinking about the graph she could have a visual representation that the two sides of the equation are not equal. Thinking about it graphically is another representation of the of the system of equations.*

*--This does build on the work the student shows. The student is almost there but only needs a little help making the final connection.*

*--Knowing that the two lines will never intersect because they have the same slope is what i hoped the student would also notice.*

*--The student was already very close and was just one step away from having a complete answer. she even said that there was in inequality. it just look a little coaxing to get the final conclusion out of the student.*

**Characteristic #2: Draws on and is consistent with the student thinking presented**

**Common Errors in Response to Zander’s Solution**

These examples highlight how PSTs might provide a response that does not focus on student thinking. Furthermore some PSTs push the student toward their way of thinking rather than build on what the student had done.

Student 5 Response

*As my first response to the student I would respond saying, “if you didn’t get stuck can you explain to me what you would expect as a solution?” I would anticipate the student to respond saying that he would get an answer like x equals some number. I would then ask the student, “ok, so what does that number mean?” I would expect the student to say that that number would be the solution to the problem. “Can you come up with a different way, other than the work that you got stuck on, to explain what that number means? Are there any other ways you have learned in class?” If the student is still having difficulty understanding how to think about this differently, I would ask the student, “is there a way that you can guess and check to find the answer? Once you find it, can you explain why some numbers worked, and others didn’t?”*

Student 6 Response

*Clarification on Zander’s thinking is necessary at this point, I’m not one hundred percent sure where this student stands. Firstly I would want to ask him if he can solve this subject with a table. Leading with a table as opposed to a graph, because at this point I am not certain that Zander could do this graphically. After he has figured this out with a table, I would lead him back to the beginning of the question. Although this is more leading than I would want to be in this situation I feel it is necessary to ask him if it is possible to move the x before he ends up with crazy fractions.*

**Exemplars in Response to Trisha’s Solution**

These examples show PST responses that build on the student’s work and the thinking evident in the solution as presented by the student.

Student 7 Response

*I would ask the student two questions. Firstly, I would ask her what getting something “obvious” really means. Then, I would ask her to plug in some more values of x. She was able to say that x=1, but she was very unsure of this. I think that if she plugged in a few different x terms, she would begin to see that there are infinitely many solutions to this system of equations.*

Student 8 Response

*I would start by having her plug one into the equation to see if her guess is valid. Then I would prompt her by saying, “Would any other numbers work for x?” After she realizes that more numbers would work, I would ask, “At the end you had 6x=6x and both sides are the same. What does this mean?” Next, I would have her graph the equations and hopefully she would see that they are the same lines and that all real numbers are solutions.*

**Characteristic #3: Draws on and is consistent with research on students' mathematical development**

**Common Error’s in Response to Mark’s Solution**

The Huntley article that the PSTs read discussed how students do not always understand how to interpret solutions that deviate from the *x*= a real number format resulting from equations with one real solution. The examples below show how PSTs did not always take this research into consideration in their responses build on the research they read in their coursework.

Student 9 Response

*I would wish to ask questions to Mark to help me understand his thinking. First, I would ask “Can you think of another method of solving that would work?” that would hopefully prompt him to go back to the original statement and try solving it symbolically using different steps (distributing the coefficient first). This may cause Mark to recognize his error in the first attempt, then I would know that it was a simple calculation mistake. If this doesn’t provide any new information, I would ask “Does your answer seem reasonable? Why or why not?” to have Mark describe the meaning behind his answer, x = 0. If he is able to explain what it means in terms of a solution and the connection between the two equations on either side of the original statement, then he has a pretty good grasp on the material. If not, we will have other issues to deal with since he doesn’t understand the concept of solving this problem.*

Student 10 Response

*The first thing that I would want to understand about the student’s thought process is why the student on one side of the equality divided the zero by zero to get a coefficient of one and on the other side of the equation it is zero. To understand this, I would ask the student, “I see you divided by zero, if you considered something divided by zero (a simpler problem) what would happen?” I would then anticipate the student to work out that a number divided by zero is zero and when a coefficient is multiplied by zero it is zero. I would then ask the question, “But does zero work as a solution?” The student would then check the answer which would prove to be a correct solution. I would respond by saying, “that’s interesting, even though the symbolic algebra is wrong, your answer still worked. How do you think that happened?”*

**Exemplars in Response to Zander’s Solution**

These examples show how two PSTs built on research from the Huntley article in their response.

Student 11 Response

*In response to Zander’s work, I would ask:*

* *Is there a way we can still get our x’s together even though we now have a fraction on one side?*
* *Why did you decide to divide by two in your second step of solving?*
* *Is there another approach you could have taken during your second step of solving if you did not want to work with fractions?*
* *When you say “get rid of them”, what do you mean?*
* *What are you doing when you are solving for x? Why do we even solve for x?*

Student 12 Response

 *–What does getting x by itself mean? There should only be one x-term and it should be on one side of the equation all alone. – This will help the student get x by itself ad see more clearly what x is.*

**Characteristic #4: Proposed interaction with student leaves space for student's future thinking**

**Common Error’s in Response to Mark’s Solution**

This example shows how PSTs might fall short of this final characteristic by taking over or re-directing student thinking.

Student 13 Response

*If he really thinks that it is appropriate to divide by zero, this is an excellent opportunity to explain that it is not possible to divide by zero. Otherwise, I need to help Mark see that zero is an answer, but it is not the only answer. I would ask him to begin again, and to stop after each step in his process so that he can think about what he has left.*

**Exemplars in Response to Mark’s Solution**

This PST provides two questions in response to the student work that encourage the student to consider what they did to solve the problem and extends the student’s thinking beyond their current procedural approach.

Student 14 Response

*To more fully understand Mark’s work and thought process, I would want to ask him why he suggests dividing by 0, and whether we can ever divide by 0. I would want to ask him this because he states, inquisitively, “you can just divide 0 from 0, so x = 0?”. I want to spark in Mark’s mind the fact that we never divide by 0, but that he does have on his paper 0\*x=0, which produces 0 = 0 because x just symbolically represents an unknown number, and 0 multiplied by any number produces 0. Similarly, another response I would give to Mark would be ‘you tell me if x=0, how can we find out if x=0?’. I feel that this question would prompt Mark to try plugging in 0 to see if it is a viable solution; I would follow this up by asking him to try another number for x and would note that there seems to be multiple answers for x. This response might lead into what I would next ask: I would also want to ask Mark what it means to have two equations equal to one another, thinking that this prompt might lead us into a discussion about graphing the two equations.*